Critical zone of the branching crack model for earthquakes: inherent randomness, earthquake predictability, and precursor modelling

Jiancang Zhuang[1] Mitsuhiro Matsu'ura[1] Peng Han[2]

[1] Institute of Statistical Mathematics, Tokyo, Japan
[2] Southern University of Science and Technology, Shenzhen, China

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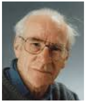
Vere-Jones' branching crack model

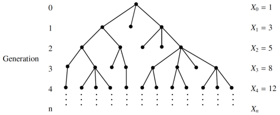
Earthquake process at micro scale.



Assumptions

1. No assumption for eqk rupture geometry.
2. Basic element of eqk rupture is crack: a micro-fracture, or a small tangential slip on a small patch of eqk fault.
3. Each crack triggers cracks nearby on the fault independently in probability.
4. Rupture process of an earthquake starts from a single crack and develops into an earthquake.
5. Galton-Watson process in mathematics.

Mathematical form



- 1st generation has only one ancestor, i.e., $Y_0 = 1$;
- Number of descendants in the $(n+1)$th generation is the total number of direct offspring from each member of the nth generation,

Criticality ν

Average number of direct offspring from one ancestor.

- - $\nu < 1$, family tree extinguishes quickly;
  - $\nu = 1$, family tree extinguishes slowly;
  - $\nu > 1$, possibly, population explodes.

Total Energy

It is reasonable to assume the energy E of the earthquake can be emulated by the total number X of cracks in VJ's branching model.

(***Dwass theorem***) For a general branching process with a single time-0 ancestor and offspring distribution Y and total population size X:

$$\Pr\{X = k\} = \frac{1}{k}\Pr\{Y_1 + Y_2 + \cdots + Y_k = k - 1\}$$

where $Y_1, Y_2, \dots, Y_k$ are independent copies of X.

Using the Dwass theorem and central limit theorem, when we can prove

$$\Pr\{X = n\} \sim \frac{1}{n\sqrt{2\pi n\sigma^2}} \exp\left[-\frac{(n(1-\mu)-1)^2}{2n\sigma^2}\right]$$

$$\sim n^{-\frac{3}{2}} \exp\left[-\frac{(n(1-\mu)-1)^2}{2n\sigma^2}\right]$$

Total Energy

$$\Pr\{X=n\} \sim n^{-\frac{3}{2}} \exp\left[-\frac{(n(1-\mu)-1)^2}{2n\sigma^2}\right]$$

When $\mu<1$, $\Pr\{X=n\} \sim n^{-\frac{3}{2}} \exp\left[-\frac{(1-\mu)^2}{2\sigma^2} n\right]$ (subcritical regime)

When $\mu=1$, $\Pr\{X=n\} \sim n^{-\frac{3}{2}}$ (critical regime)

When $\mu<1$ (subcritical regime),

$$\Pr\{\text{magnitude} \geq m\} \sim \text{const} \cdot 10^{-0.75m - c\,10^{-d\,m}}$$

$$\Pr\{\text{moment} \geq M\} \sim \text{const} \cdot M^{-0.5} \exp(-M/M_C)$$ (tapered Pareto)

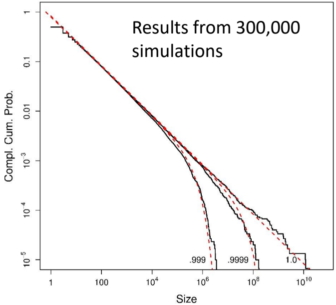
When $\mu=1$ (critical regime),

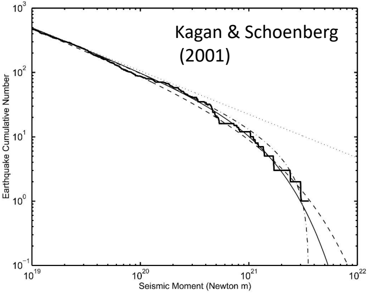
$$\Pr\{\text{magnitude} \geq m\} \sim \text{const} \cdot 10^{-0.75m}$$ i.e., $b = 0.75$

$$\Pr\{\text{moment} \geq M\} \sim \text{const} \cdot M^{-0.5}$$ (Pareto, G-R law)

M_C: corner magnitude c, d: constants

Simulation of G-R law





μ <1 (subcritical regime) : $\Pr\{\text{moment} \ge M\} \sim \text{const} \cdot M^{-0.5} \exp(-M/M_C)$(tapered Pareto)

μ =1 (critical regime), $\Pr\{\text{moment} \ge M\} \sim \text{const} \cdot M^{-0.5}$ (Pareto, G-R law)

M_C: corner magnitude c, d: constants

Complementary cumulative distributions for simulated seismic moments (c.f. *Zhuang et al.* 2006).

Total Energy (3): Why $b = 0.75$? Is it universal?

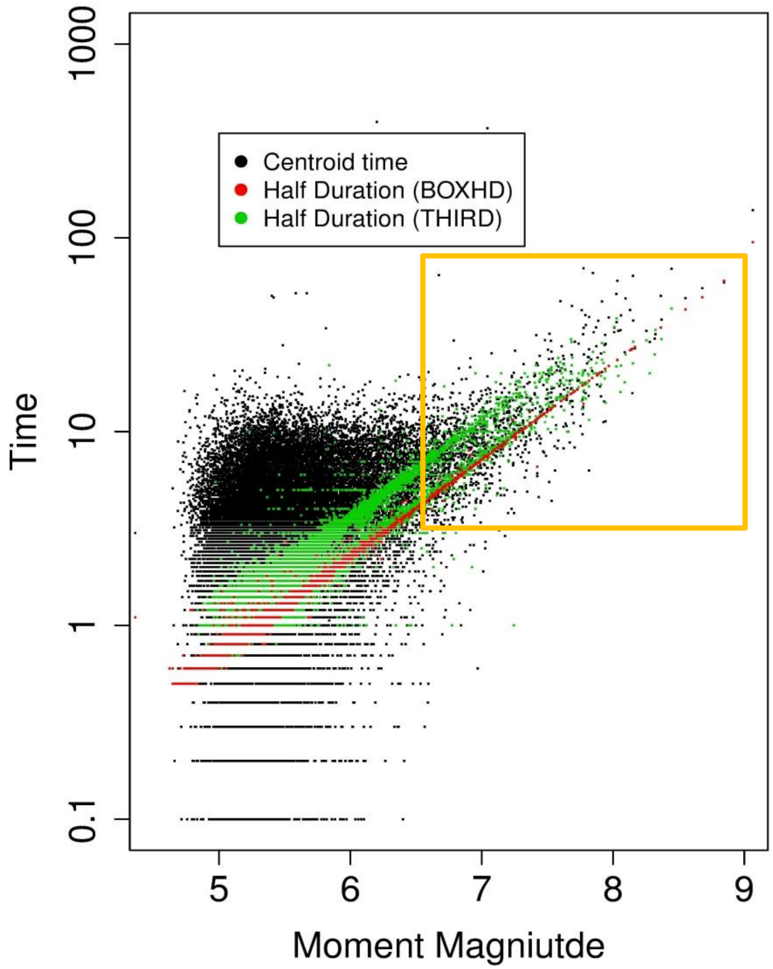
Kagan (2010, Tectonophysics): ***Yes***.

He also listed several reasons why observed b is higher than 0.75.

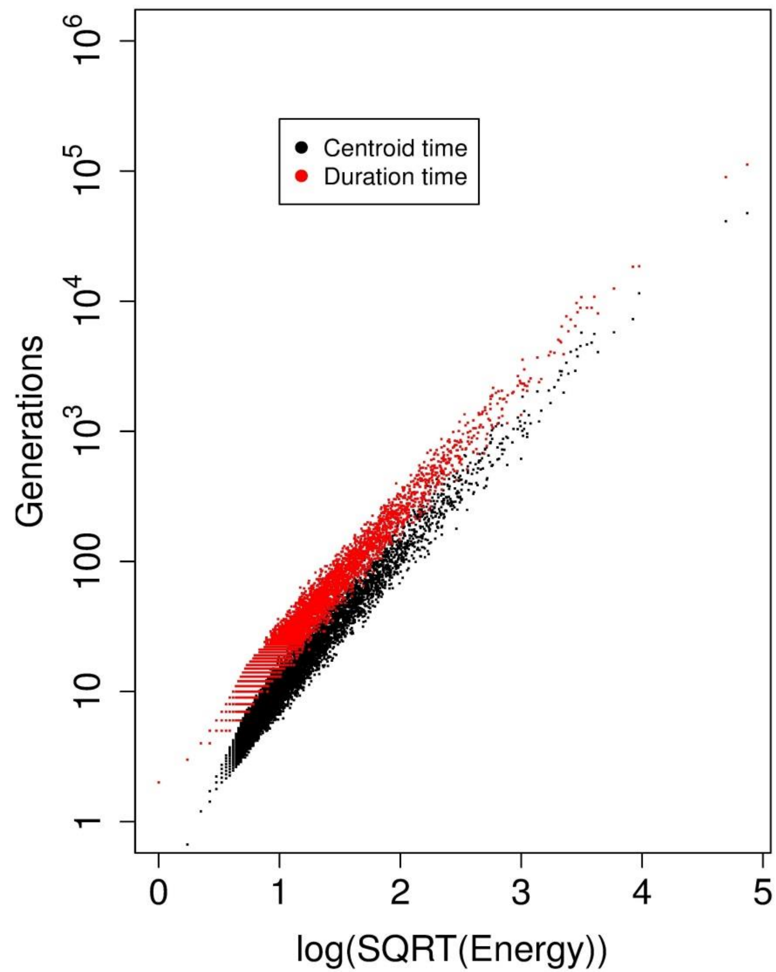
- Inappropriate usage of magnitude scales other than moment magnitude.
- Ignorance of the maximum or corner moment M_C.
- Mixing EQs from tectonics settings that have different corner magnitude.
- Relative seismic moment errors increase when decreasing EQ size, resulting higher b value.
- Using tensor sum (cscalar) instead of moment tensor solution.
- Corner magnitude is likely to change with depth.

Distribution of duration time and relation to energy

Global CMT



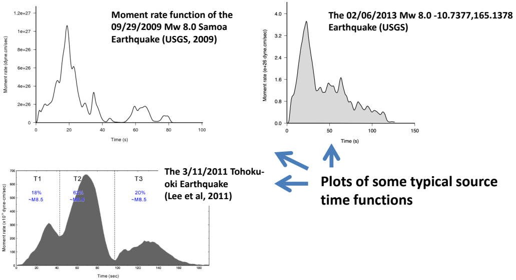
Simulation



What EQK features can branching crack model simulate?

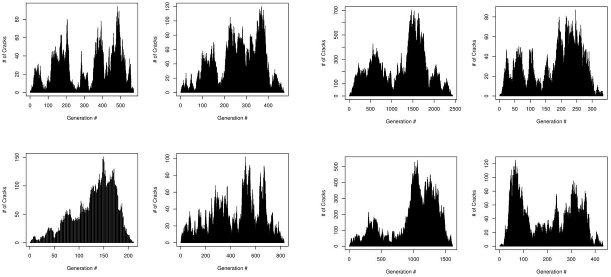
Source-time function

- Each generation is a time step;
- Earthquake energy at each time step is proportional to total number of cracks in that generation.



Plots of some typical source time functions

What EQK features can branching crack model simulate?



Simulated source time functions for large events

Temporal distribution of energies (source-time function)

1. Regarding a "generation" as a time step, VJ's branching crack model has similar source time functions as earthquakes.
2. If the branching process does not stop at a certain time step, any number of cracks are possible to be produced in its continuation.
3. This randomness of VJ's branching model explains why the EQ magnitude cannot be determined unless the recorded waveforms contain information of the whole source process.

What EQK features can branching crack model simulate?

Source-time functions imply *Inherent Randomness.*

- If the branching crack process does not stop, any number of cracks or peaks are possible to be produced in its continuation.
- Can earthquake magnitude be determined until eqk rupture process finishes? *No.*
- Can earthquake magnitude be predicted?

Critical zone

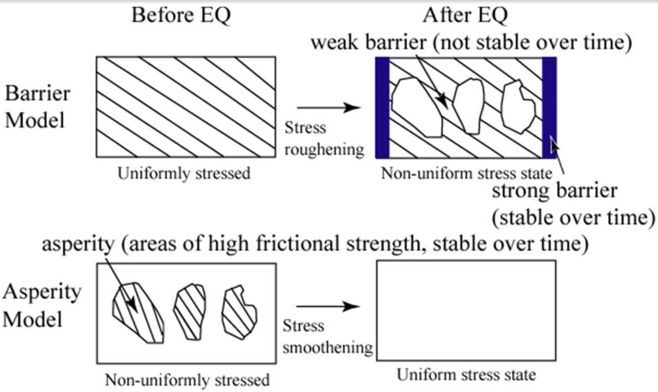
Difference between seismogeneric medium and ideal model

- Ideal model: homogeneous, critical everywhere.
- Seismogeneric medium: inhomogeneous, critical zone.

Critical zone (2)

Two types of critical zones

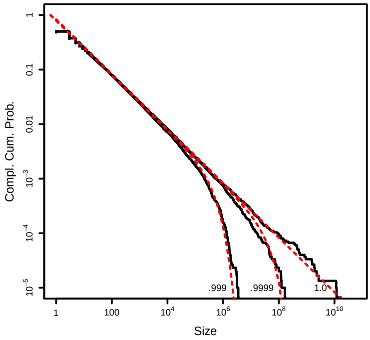
- Barrier model: same stress level, surrounding material has higher strength (Aki, 1984)
- Asperity model: accumulated stresses in source area released by previous earthquakes and slow slips (Lay & Kanemori, 1981)



Determining critical zone and critical state

Corner magnitude, *b*-value, and activation of micro-seismicity

- Increasing M_r:
  break of sharp pencil lead (crack): 10^{-9} J
  $\rightarrow$ corner magnitude: -2.5
- Decreasing *b*-value:
- Increasing rate of small events:
  AMR phenomena



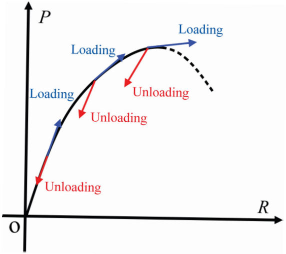
Determining critical zone and critical state

Stress field – LURR (load/unload response ratio)

- LURR (*Yin et al*)

$$Y = \frac{\Delta R_+/\Delta P_+}{\Delta R_-/\Delta P_-},$$

- Loading response rate $\Delta R_+/\Delta P_+$; unloading response rate $\Delta R_-/\Delta P_-$.
- $Y = 1$: material in the elastic phase; $Y > 1$: damage formation phase; $Y \to \infty$: the failure stage



Determining critical zone and critical state

ETAS model: Criticality in EQK clusters

- ETAS model (*Ogata*, 1988)

$$\lambda(t) = \frac{\mathbf{E}\left[N(\mathrm{d}t) \mid \mathcal{H}_t\right]}{\mathrm{d}t} = \mu + \sum_{i:t_i<t} \kappa(m_i) g(t-t_i),$$

- $\kappa(m) = A\exp[-\alpha m]$: Utsu-Seki law
- $g(t) = (p-1)(1+t/c)^{-p}/c$: Omori-Utus formula
- Criticality of ETAS: $\rho_{\mathrm{etas}} =< \kappa(m) >$

ETAS model based anomalies

- higher background seismicity;
- higher clustering effect (higher ρ_{etas}, foreshocks?).

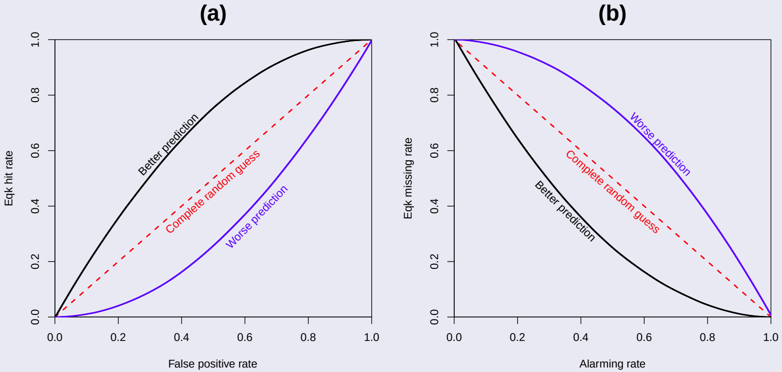
Determining critical zone and critical state

Changes in deformation, gravity and electromagnetic field

- Surface deformation shows the strain of the upper crust
- Changes of gravity field: information about the motion of crustal medium in the vertical direction. (e.g., Chen et al. 2015)
- Radiative signals from the geo-electromagnetic field (e.g., Zhuang, 2015: Du et al, 2012; Hang et al., 2014, 2017).
- All above are not so directly linked to the criticality of seismogeneric zones, and thus might not be as effective as the seismicity- or stress-based indicators.

ROC (Receiver Operating Characteristic) curve and Molchan error diagram

Systematic scheme for verifying the precursory effect of anomalies



- Wang et al. 2012; Chen et al. 2015; Han et al., 2017

Verification of earthquake precursors and performance quantification

Probability gain

$$p_g = \frac{P(EQK \mid Anomaly)}{P(EQK)} \tag{3}$$

- Probability gains obtained in past studies for precursors are low, usually 2 to 4.
- Probability gains for ETAS model up to a scale 10 to 10^3 (e.g., *Helmstetter et al.*, 2006; *Zhuang*, 2011).

Modelling strategies

Hawkes' self- and external-exciting (Linlin) model

(Hawkes, 1972,9174; Ogata et al., 1988; Ma & Vere-Jones, 2000)

$$\lambda(t) = \mu + \sum_{i:\, t_i<t} g(t - t_i) + \sum_{j:\, s_j<t} h(t - s_j), \tag{4}$$

- (Zhuang et al., 2005, 2013) ULF underground electric signals and seismicity near Beijing. Probability gain ≈ 3.

Modelling strategies

An effective model

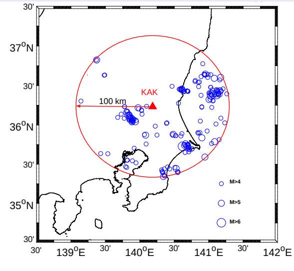
- Clustering effect is the biggest predictable component in seismicity. Model should be extension of the ETAS model.

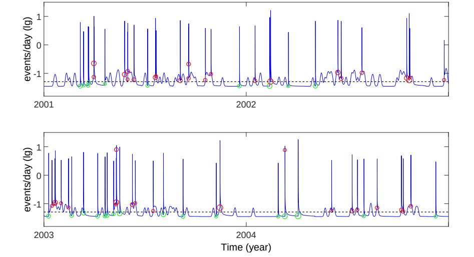
$$\lambda(t) = \mu + \sum_{i:t_i<t} \text{ETAS terms} + \sum_{j:s_j<t} B \exp\left[-\frac{(t-s_j-T)^2}{2D^2}\right],$$

  - T: center of forecasted time interval;
  - D: length of forecasted time window;
  - B: strength of explanatory effect.
- Han et al (2006): geomagnetic anomalies at Kakioka and $M \geq 4.0$ EQKs within 100 km

Modelling strategy

Geomagnetic anomalies at Kakioka and $M \geq 4.0$ EQKs within 100 km





$$\lambda(t) = \mu + \sum_{i:t_i<t} \text{ETAS terms} + \sum_{j:s_j<t} B \exp\left[-\frac{(t-s_j-T)^2}{2D^2}\right],$$

Conclusions

- Key to more reliable eqk forecast: (1) determine the size of the critical zone, (2) monitor when the system is critical.
- Size of critical zone controls maximum magnitude of future eqks.
- Critical zone can be detected by stress field and other phenomena, including seismicity- and non-seismicity-based precursors.
- Due to inherent randomness of eqk rupture process, precursors have an upper limit of probability gain.
- High performance forecasts can be and possibly can only be made based on multidisciplinary precursors.
- As clustering is biggest predictable component in seismicity, effective modelling of anomalies is ETAS with external excitations.

Conclusions (continue...)

- (optimistic). Main task is to develop monitoring technologies that can help us to detect effective precursory anomalies and to determine the size of the critical zone and the critical status of the area of interests.
- Developing statistical inference and modelling methods for multidisciplinary precursors are also indispensable.