Probability forecasts of a large earthquake by combination of statistical characteristics and anomalies of seismic activity

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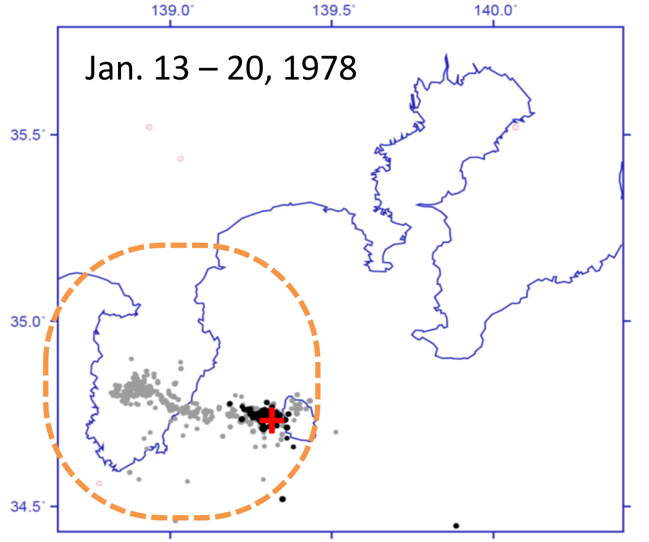
1978 (Jan.14th.12 :24)
Izu-Oshima-Kinkai eathquake of M7.0

Utsu (*Rep. Coordinating Committee for Earthquake Prediction*, 1978)



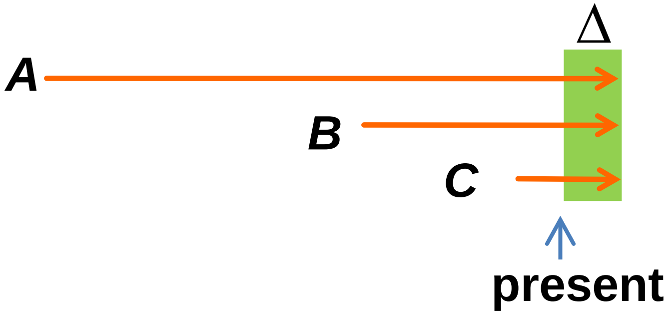
Evaluated probability of the M ≥ 6.5 earthquake

0: Secular probability of M ≥ 6.5 earthquake; (Case 1) Wider region, once per 30 years
(Case 2) Narrower region, once per 50~100years

A: Long-term anomaly = Uplift of Izu Peninsula ;
Earthquake probability = 1/3 within 5 years from uplift start

B: Intermediate-term anomalies = Radon, water table change, & volumetric strain anomalies ;
Earthquake probability = 1／10 within 1 month

C: Short-term anomaly : Seismic activity started at the west of Oshima Is. at January 14 morning;
Foreshock probability = 1／35 within 3days



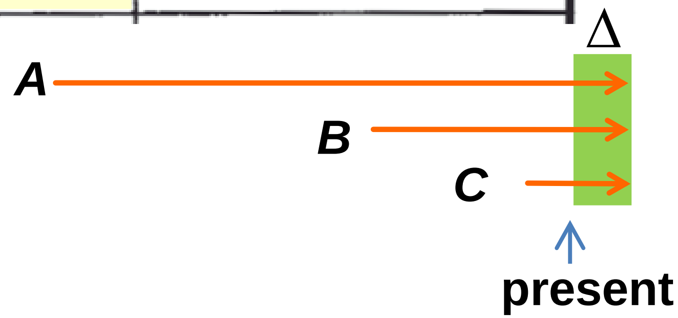
Precursors < Anomalies

Probability per each unit time Δ (Utsu, 1978)

τ		3 hours	1 day	3 days
Uplift	P_A (Long-term)	0.0000228	0.000183	0.000548
Foreshock	P_B (Short-term)	0.00119	0.00952	0.0286
strain etc	P_C (Medium)	0.000417	0.00333	0.01
$M \geq 6\frac{1}{2}$ P_O (Secular)	Case I	0.0000114	0.0000913	0.000274
	Case II	0.00000342	0.0000274	0.0000821

Utsu formula

$$P(\mathrm{M} \mid \mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = \frac{1}{1 + \left(\frac{1}{P_A} - 1\right)\left(\frac{1}{P_B} - 1\right)\left(\frac{1}{P_C} - 1\right) \Big/ \left(\frac{1}{P_0} - 1\right)^{3-1}}$$

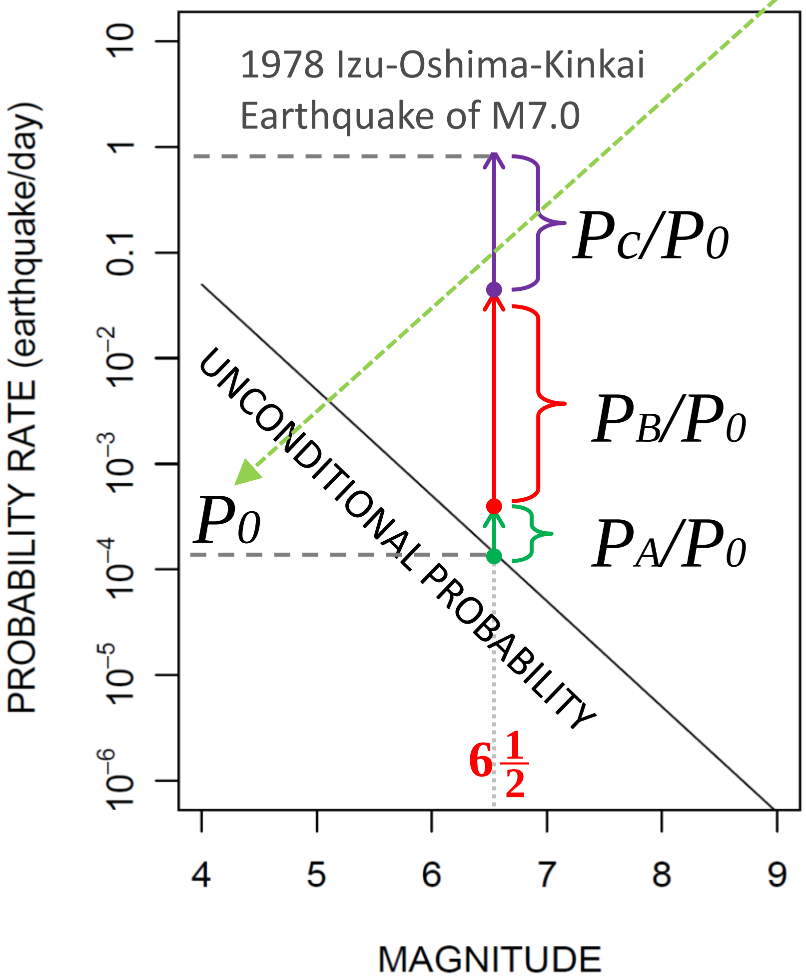


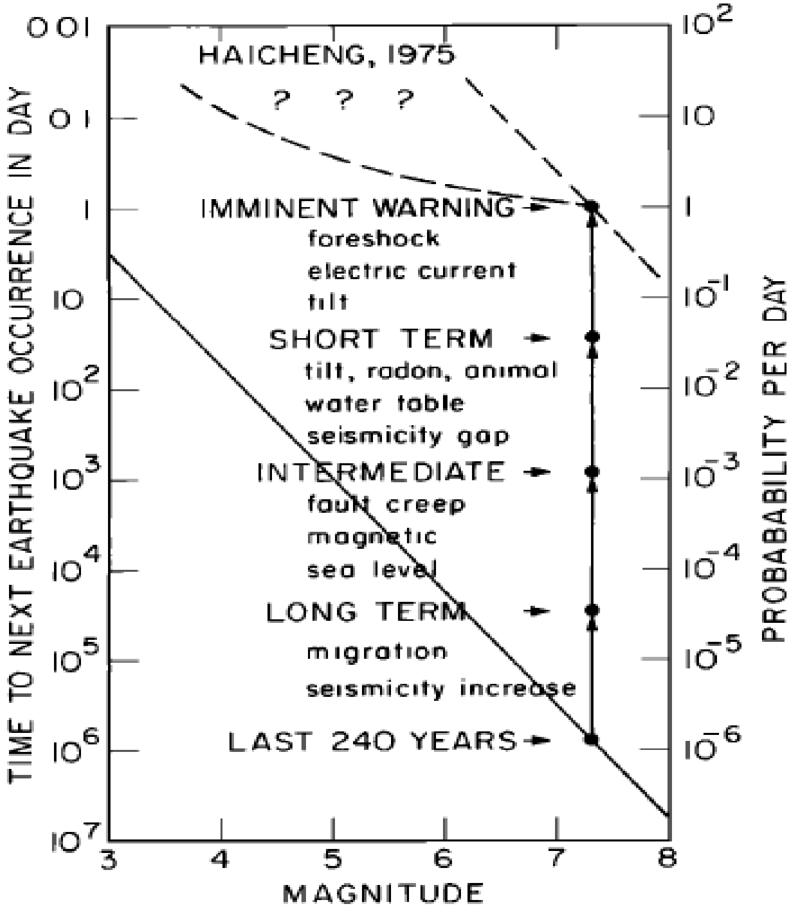
		3 hours	1 day	3 days
$P(\mathrm{M} \mid \mathrm{A} \cap \mathrm{B} \cap \mathrm{C})$	Case I	0.080 (0.011)	0.41 (0.081)	0.68 (0.21)
	Case II	0.49 (0.11)	0.89 (0.49)	0.96 (0.75)
$P(\mathrm{M} \mid \mathrm{A} \cap \mathrm{B})$	Case I	0.0024	0.019	0.056
	Case II	0.0079	0.060	0.16
$P(\mathrm{M} \mid \mathrm{B} \cap \mathrm{C})$	Case I	0.042	0.26	0.52
	Case II	0.13	0.54	0.78

$$P(M \mid A,B,C,\cdots,S) = \frac{1}{1+\left(\frac{1}{P_A}-1\right)\left(\frac{1}{P_B}-1\right)\left(\frac{1}{P_C}-1\right)\cdots\left(\frac{1}{P_S}-1\right)\Big/\left(\frac{1}{P_0}-1\right)^{N-1}} \approx P_0 \cdot \frac{P_A}{P_0}\frac{P_B}{P_0}\frac{P_C}{P_0}\cdots\frac{P_S}{P_0}$$

Aki (1981)

Probability gain = $\dfrac{\textbf{Conditional Probability based on an anomaly}}{\textbf{Baseline Probability of Large Earthquake}}$





Cao & Aki (1983) JGR

1975M7.3 Haicheng 9%/day

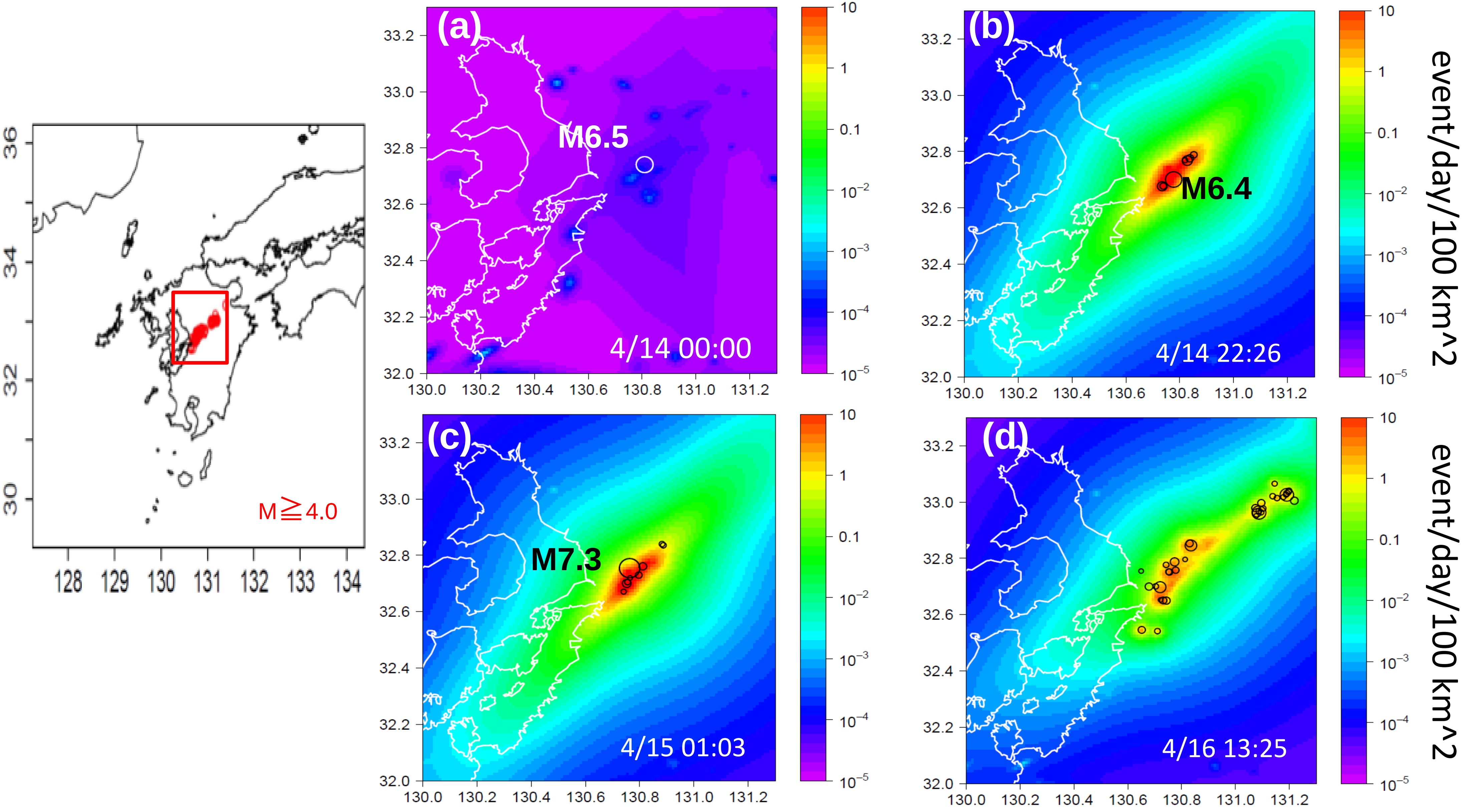
1976M7.8 Tangshan 9%/day

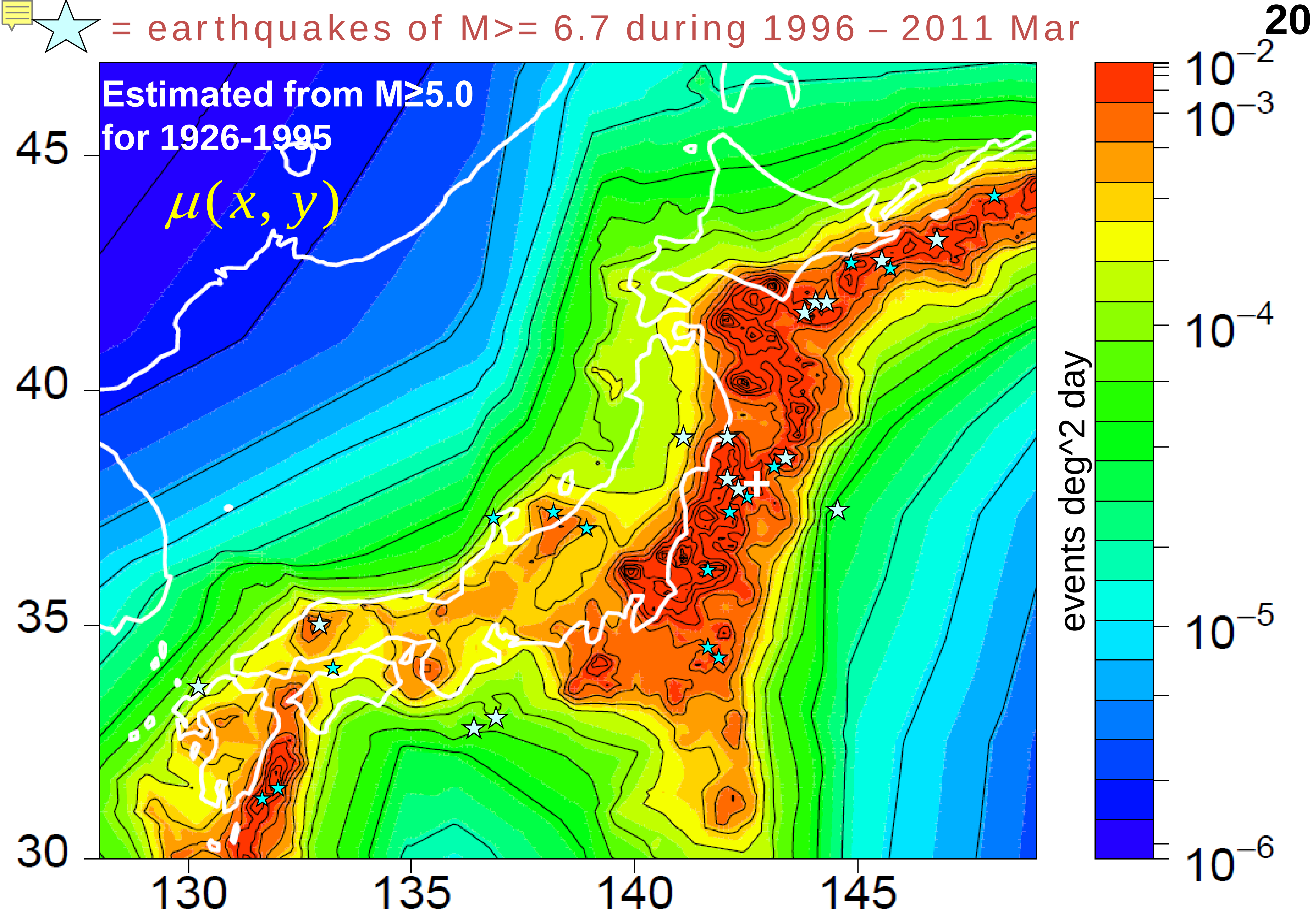
1976M7.6 Longlin 7%/day

1976M7.2 Songpan 8%/day

$$\lambda(t,x,y \mid H_t) = \mu(x,y) + \sum_{\{j;t_j<t\}} \frac{K_0(x_j,y_j)}{(t-t_j+c)^{p(x_j,y_j)}} \left[\frac{(x-x_j,y-y_j)\, S_j^{-1}\, (x-x_j,y-y_j)^t}{e^{\alpha(x_j,y_j)(M_j-M_c)}} + d \right]^{-q}$$

Rates of M≧4 event during the 2016 Kumamoto sequence

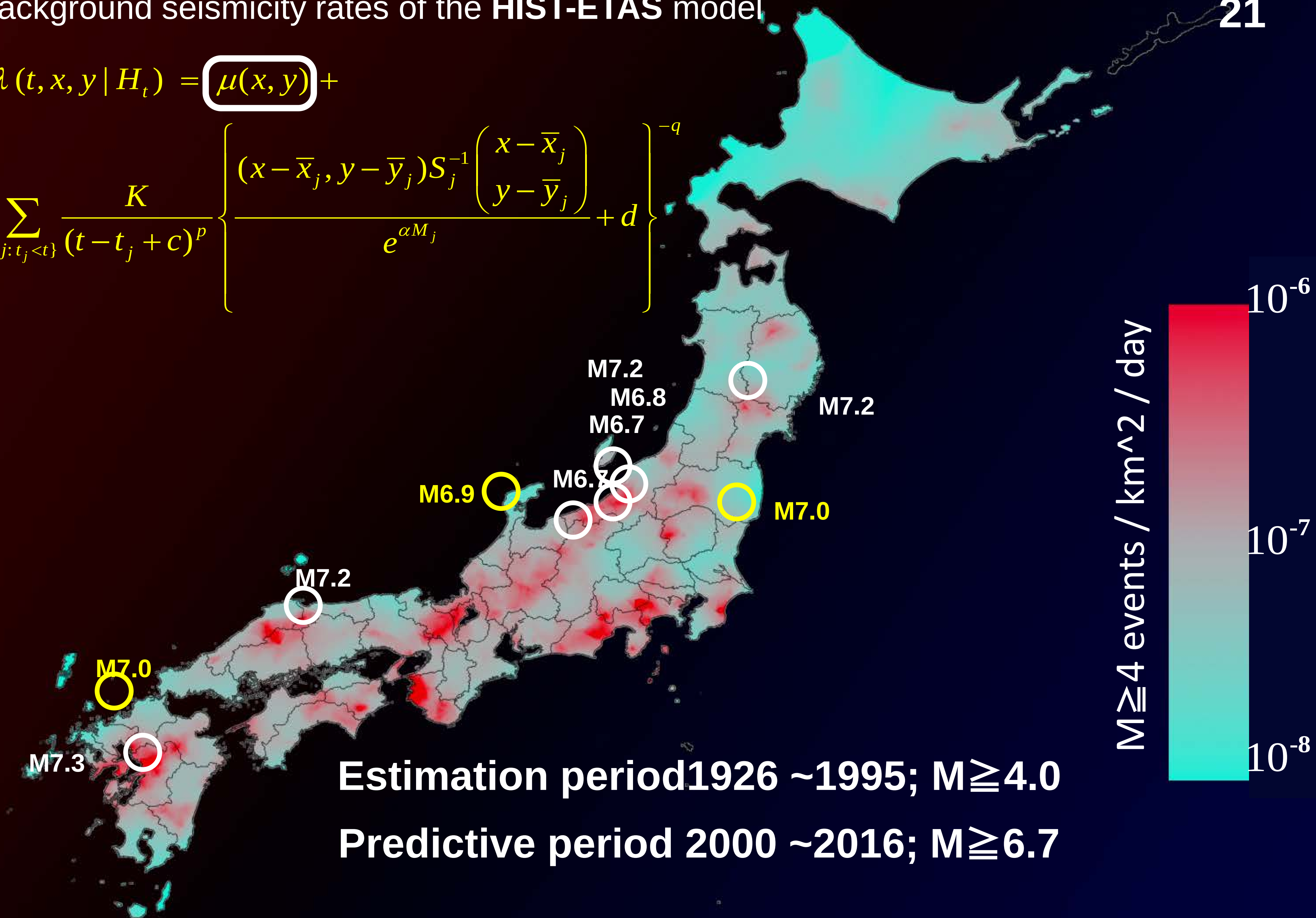




= earthquakes of M>= 6.7 during 1996 – 2011 Mar
Estimated from M≥5.0
for 1926-1995
$\mu(x, y)$
45
40
35
30
130
135
140
145
events deg^2 day
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}

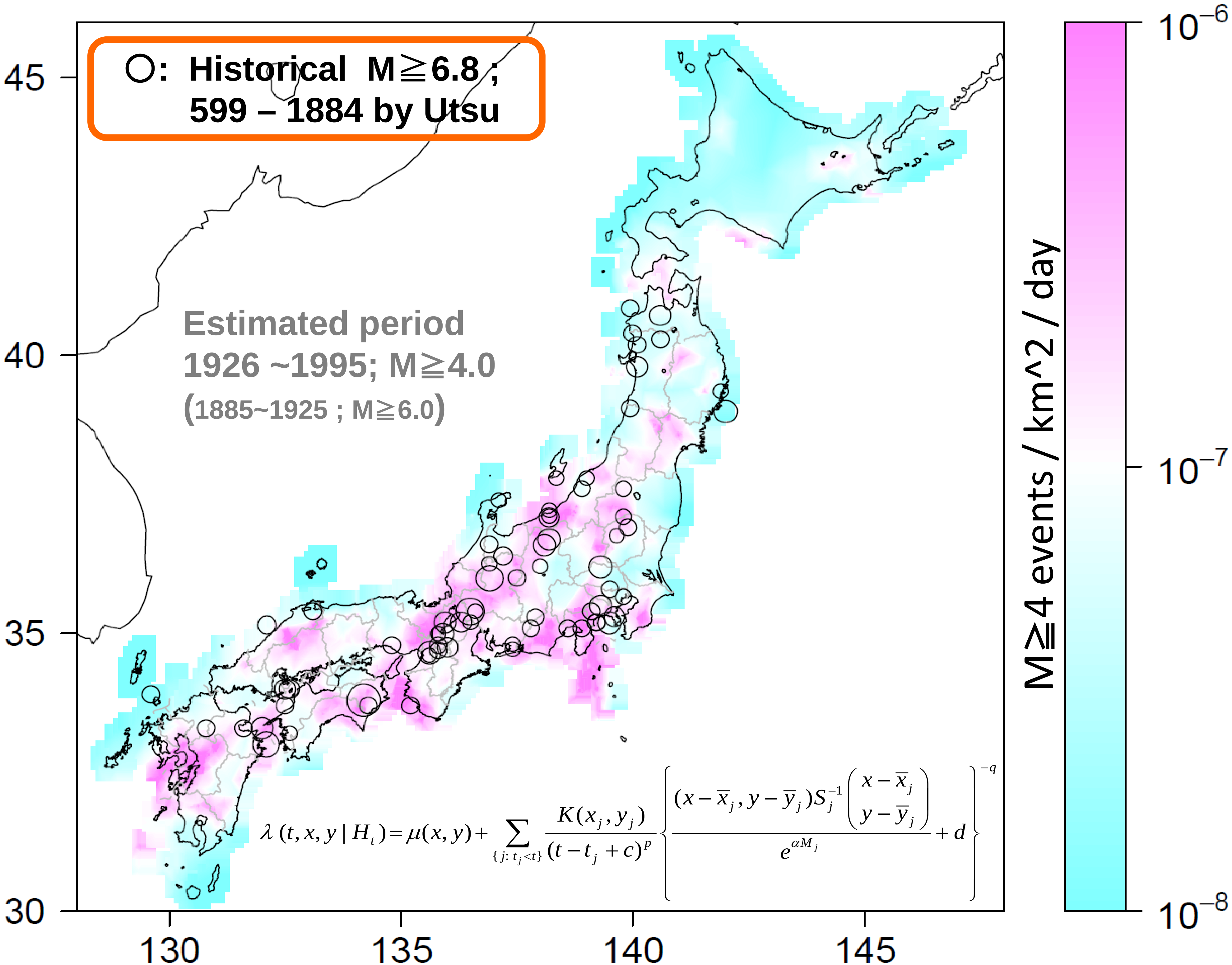
Background seismicity rates of the **HIST-ETAS** model

$$\lambda(t,x,y\,|\,H_t) = \boxed{\mu(x,y)} + \sum_{\{j:t_j<t\}} \frac{K}{(t-t_j+c)^p} \left\{ \frac{(x-\bar{x}_j, y-\bar{y}_j) S_j^{-1} \begin{pmatrix} x-\bar{x}_j \\ y-\bar{y}_j \end{pmatrix}}{e^{\alpha M_j}} + d \right\}^{-q}$$



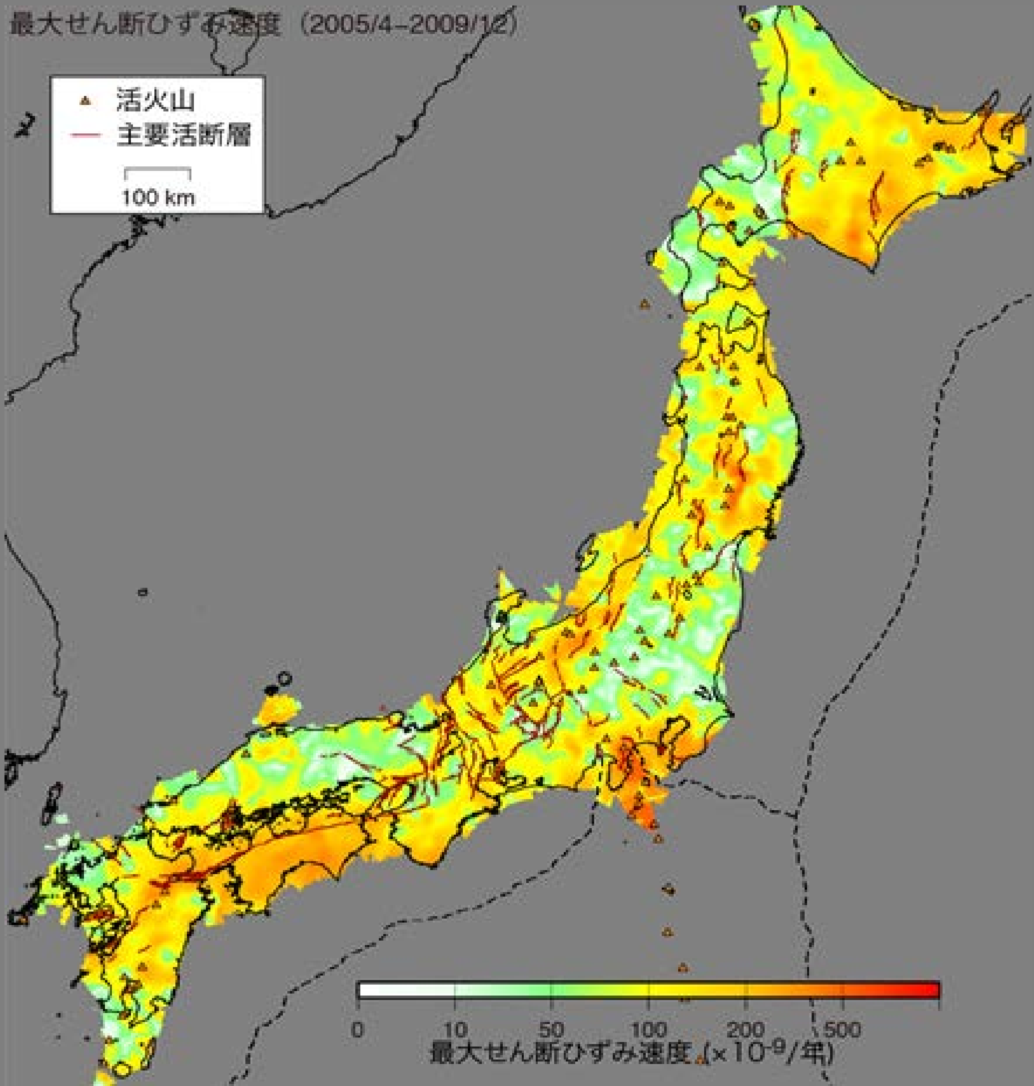
Estimation period1926 ~1995; M≧4.0

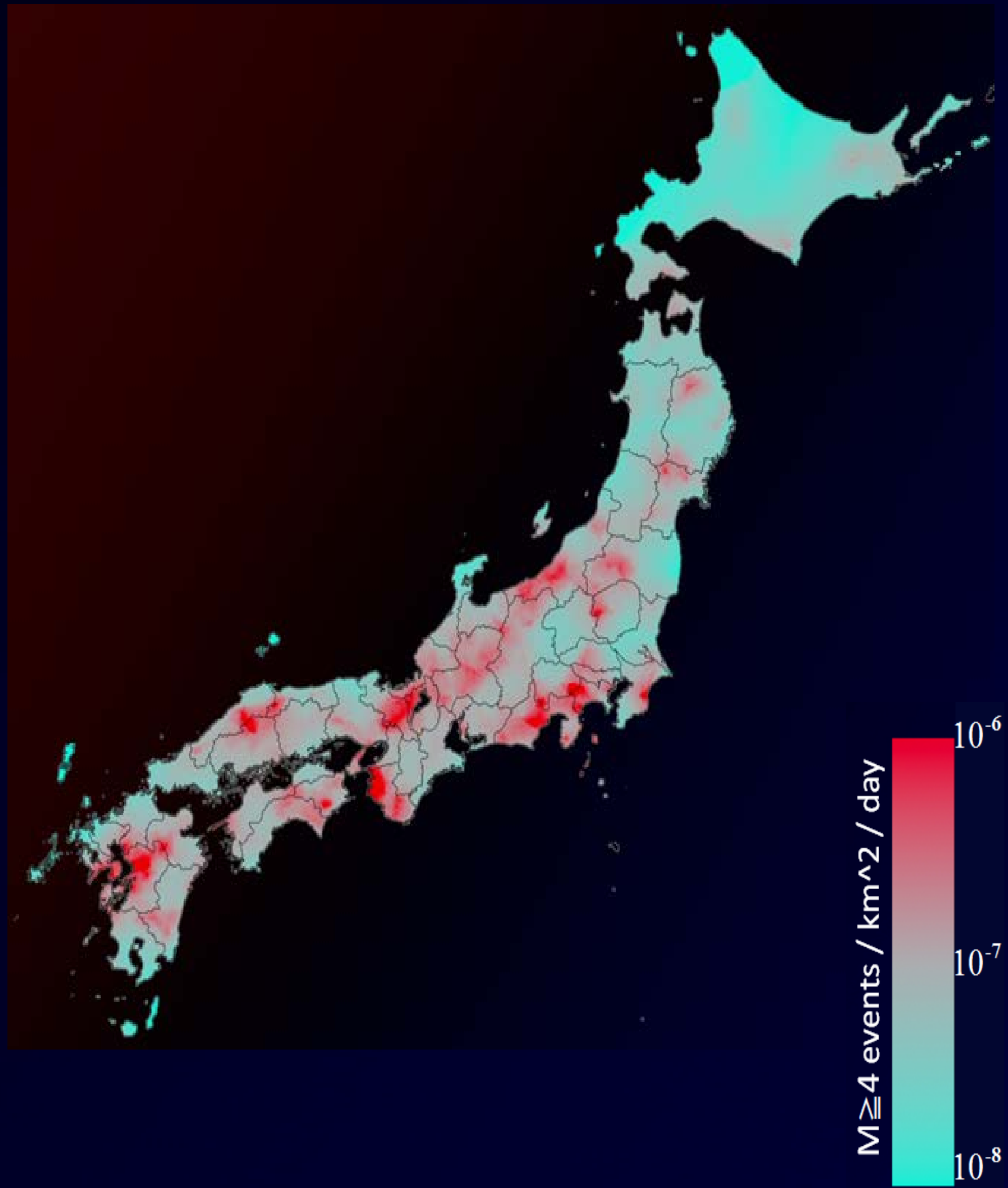
Predictive period 2000 ~2016; M≧6.7


○: Historical M≧6.8 ;
599 – 1884 by Utsu
Estimated period
1926 ~1995; M≧4.0
(1885~1925 ; M≧6.0)
$\lambda(t,x,y \mid H_t) = \mu(x,y) + \sum_{\{j:\, t_j<t\}} \frac{K(x_j,y_j)}{(t-t_j+c)^p} \left\{ \frac{(x-\bar{x}_j, y-\bar{y}_j) S_j^{-1} \begin{pmatrix} x-\bar{x}_j \\ y-\bar{y}_j \end{pmatrix}}{e^{\alpha M_j}} + d \right\}^{-q}$
M≧4 events / km^2 / day
10^{-6}
10^{-7}
10^{-8}
45
40
35
30
130
135
140
145

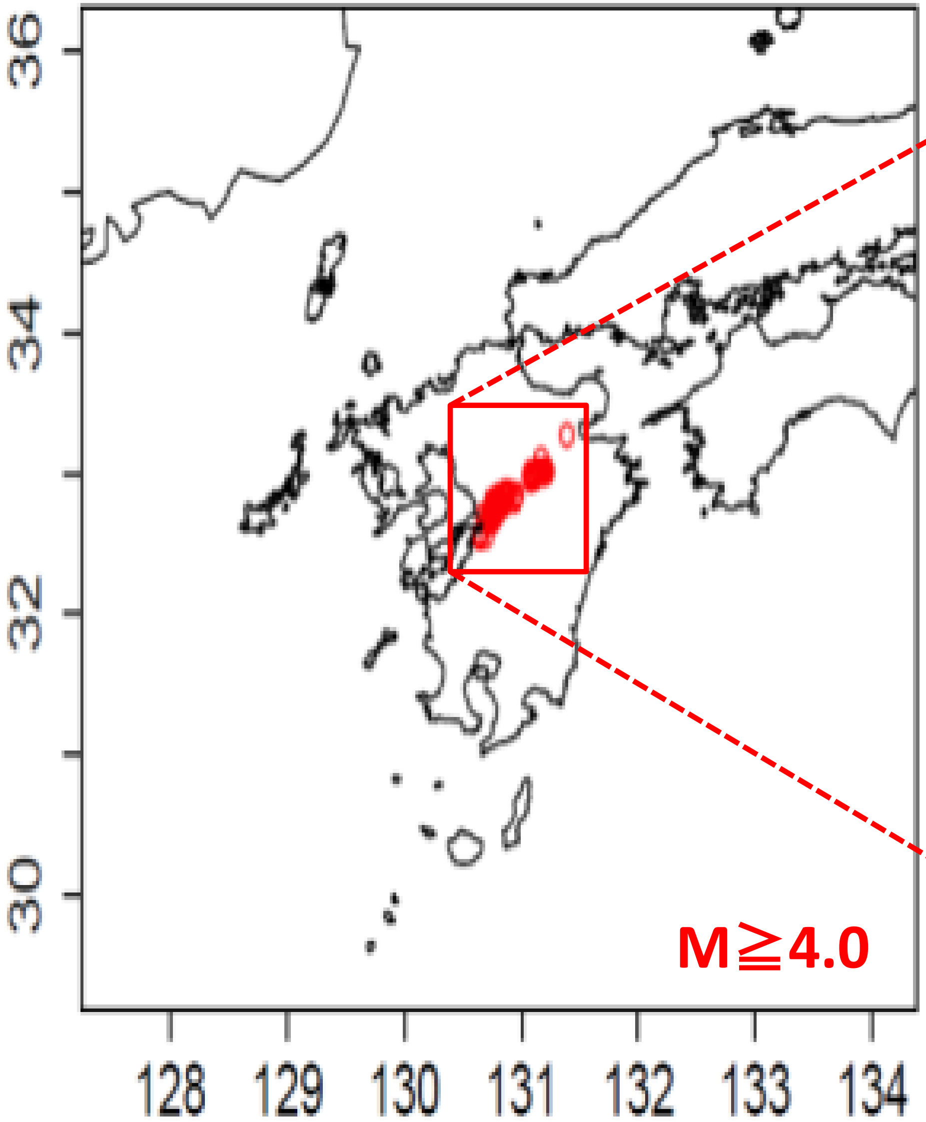
Nishimura (2017, Newton)
2005—2009 Max. Shear Strain Rate

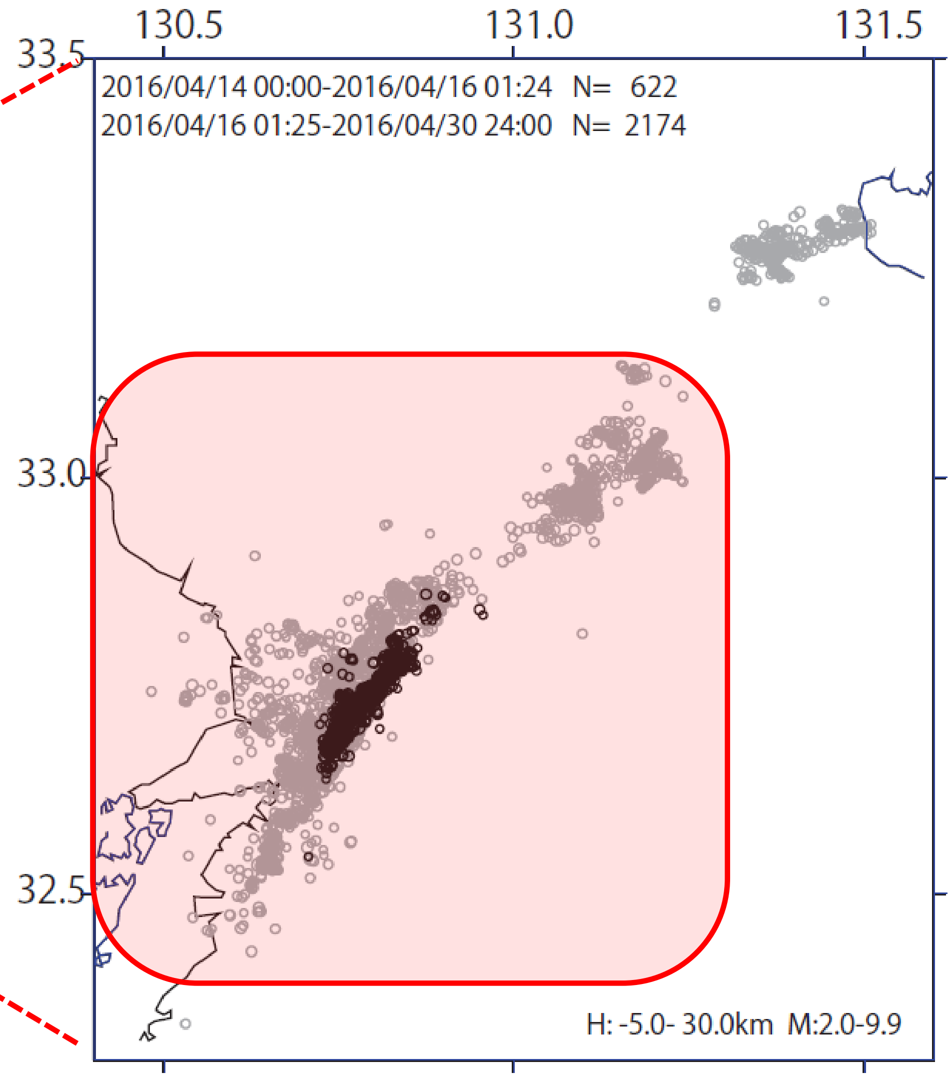
Estimation period 1926 ~1995
1885~1925 | 1926 ~1995; M≧4.0



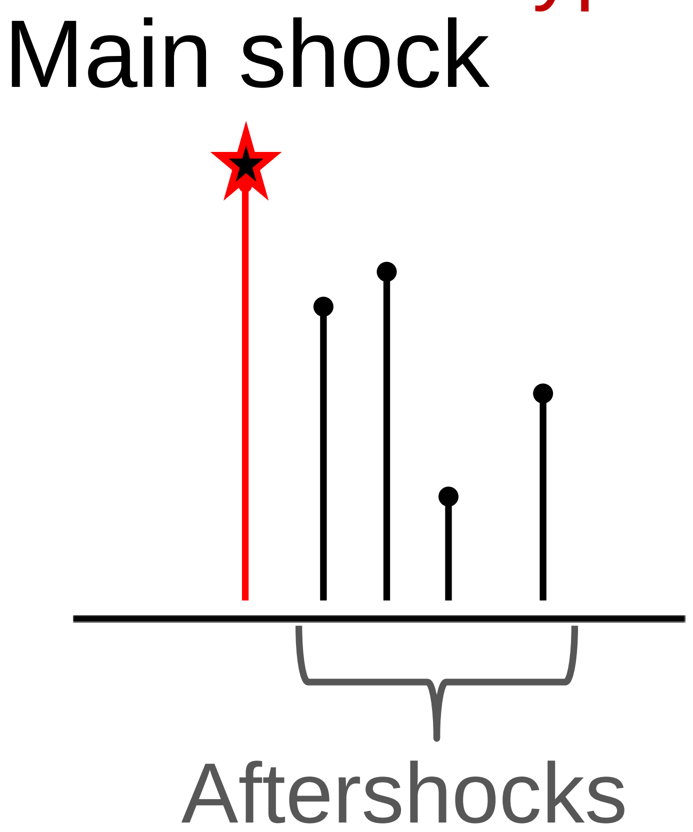


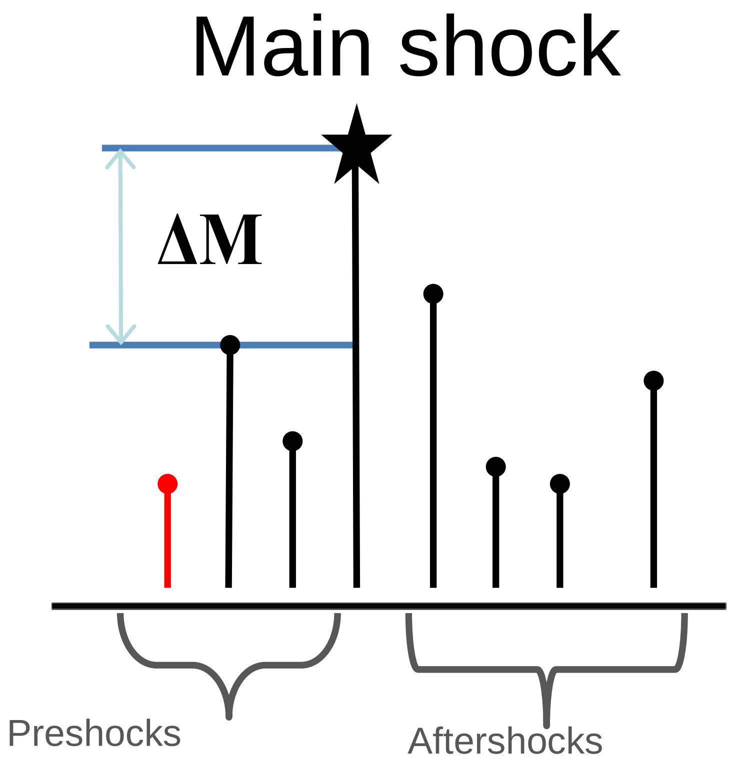
2016 Kumamoto Earthquakes（M6.5, M6.4, M7.3)





Type of clusters



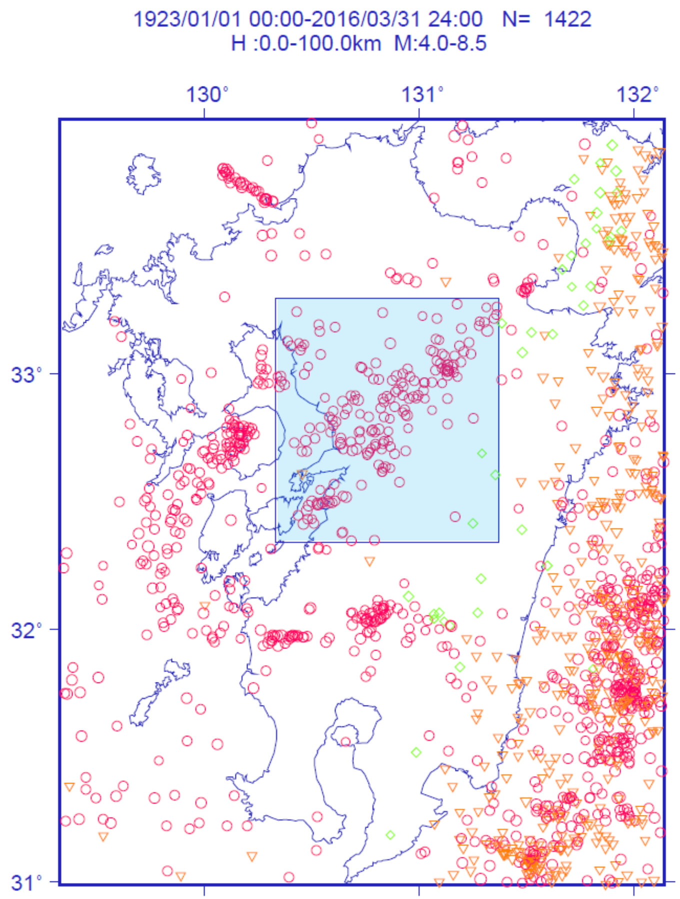


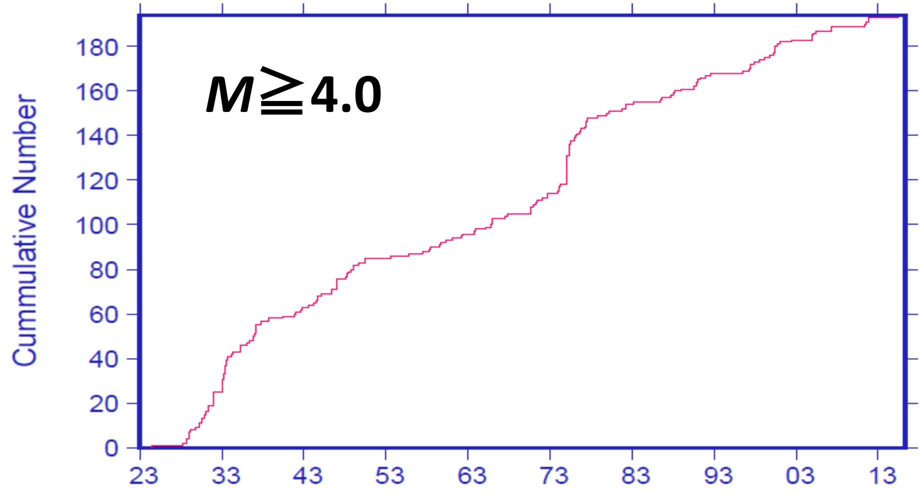
① Mainshock-Aftershock-type

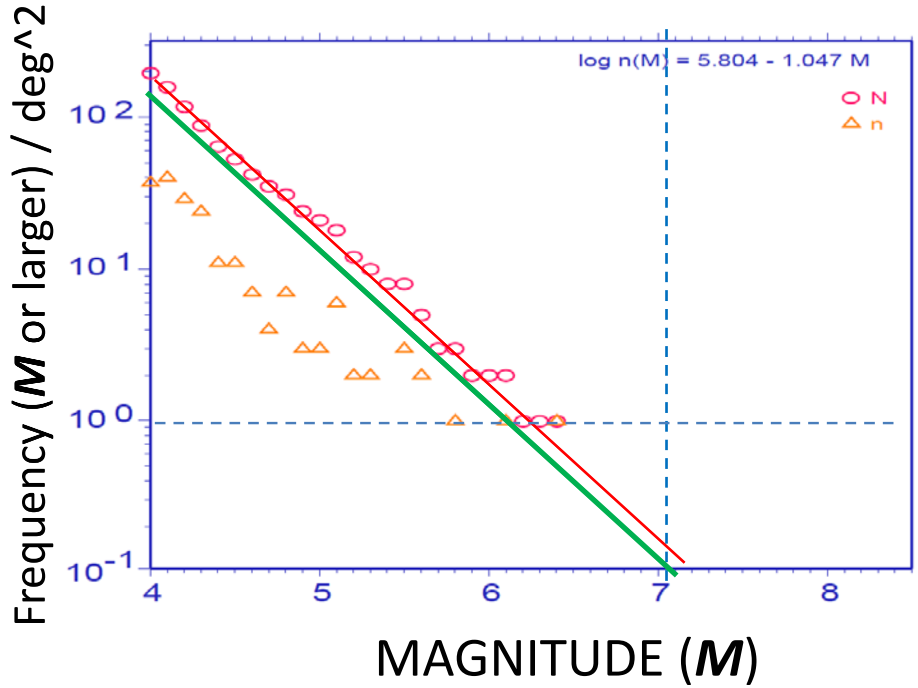
② Swarm-type $\Delta M < 0.45$

③ **Foreshock-type** $\Delta M > 0.45$

Secular probability of M≧7 earthquake in Kumamoto Area







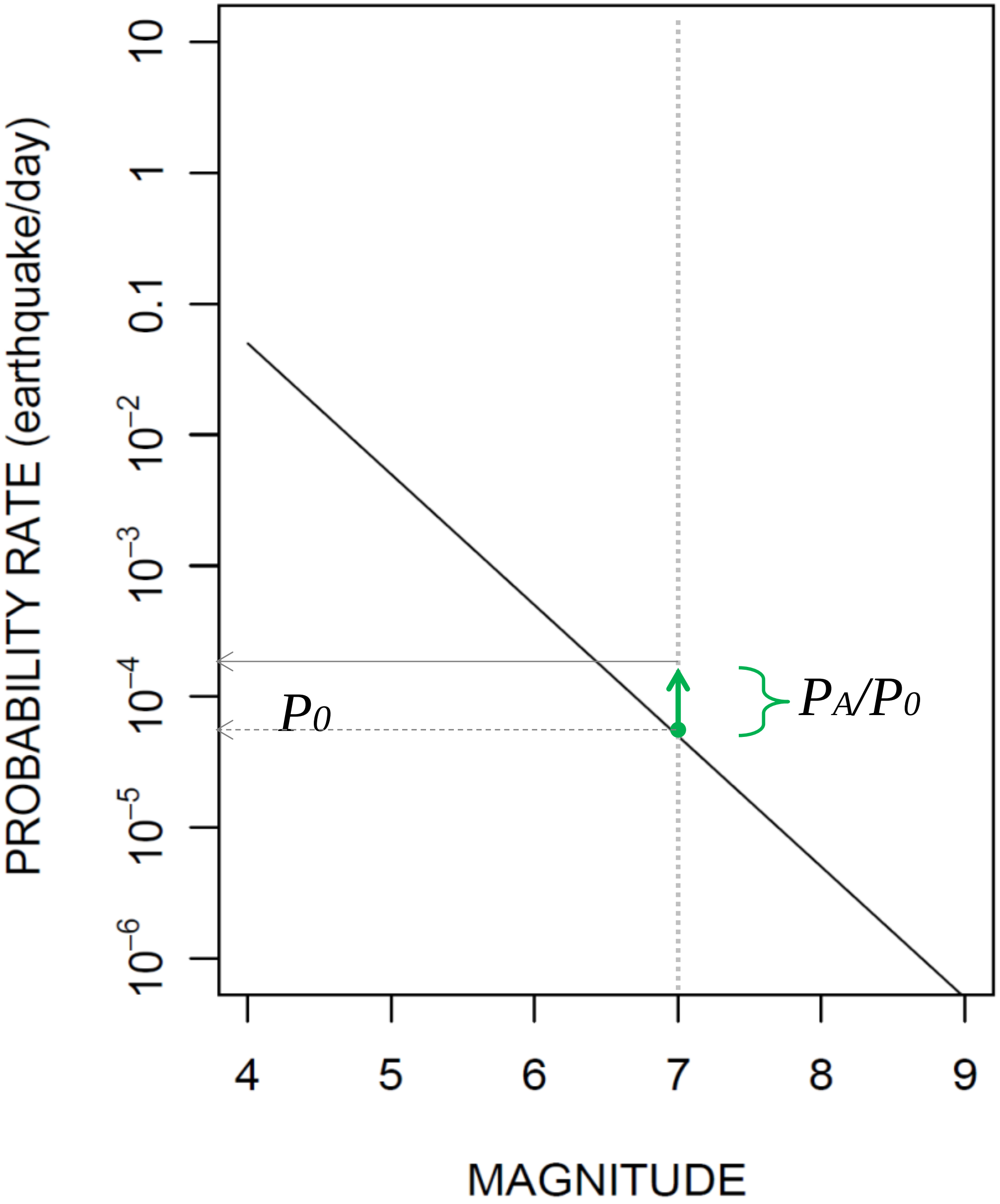
Background rate μ of the ETAS model

$$\lambda(t \mid H_t) = \mu + \sum_{\{j;\, t_j<t\}} \frac{K e^{\alpha(M_j-4.0)}}{(t-t_j+c)^p}$$

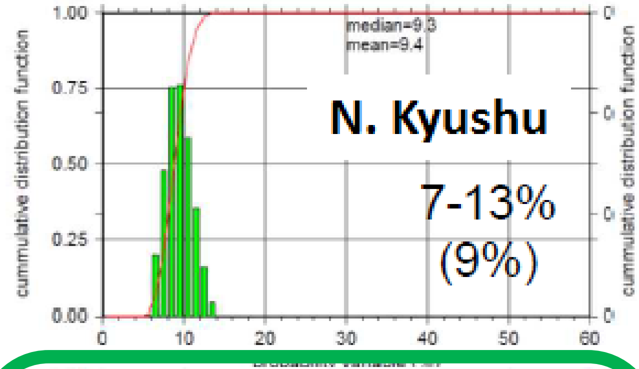
Secular probabilities of $M \geqq 7$ earthquake in Kumamoto region

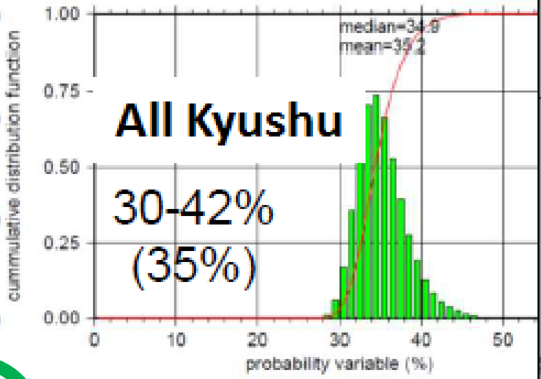
Unit time		**30yrs**	**1yr**	**1mo.**	**1day**
Secular					
S1	**ETAS-μ & G-R**				**0.00027 %**
S2	**#(M≧4) & G-R**				**0.00058 %**
Long-term					
A1	Futagawa Fault	0.9%			0.000082 %
A2	Kumamoto Dist.	10.5 %			0.00096 %
A3	Central Kyushu	21.0 %			0.0019 %
Medium-term					
B1	Triggering		0.5%		0.0014 %
B2	Quiescence		2.0%		0.0055 %
Short-term					
C	Foreshocks			5.0%	0.17 %

Long-term Probability

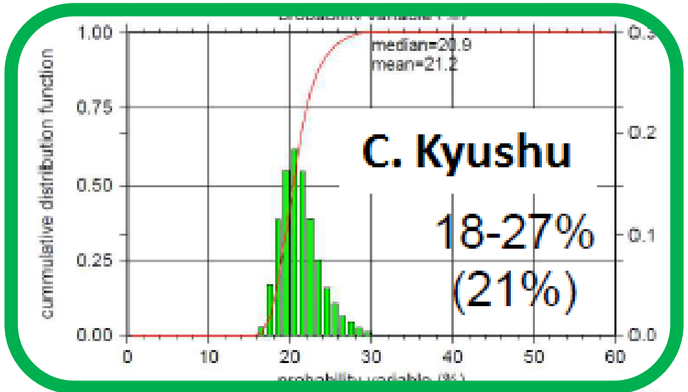


Earthquake occurrence probability of M≥6.8 occurring on active faults in each region (30 years from now) taking into account combinations of multiple active faults

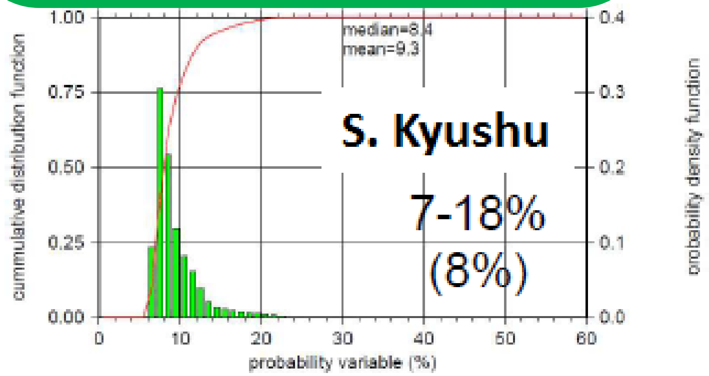


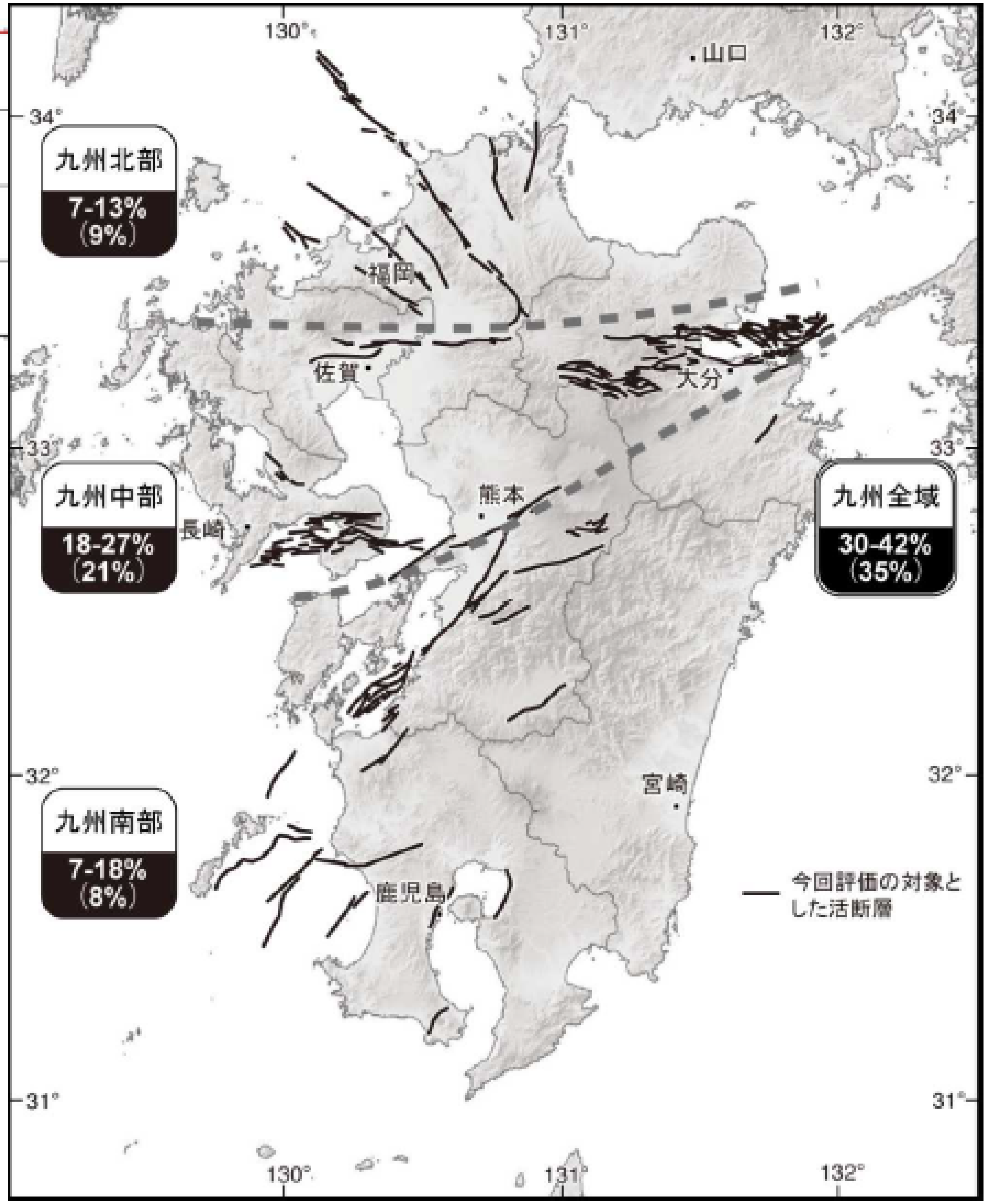


95% range
(median)



BPT simulations



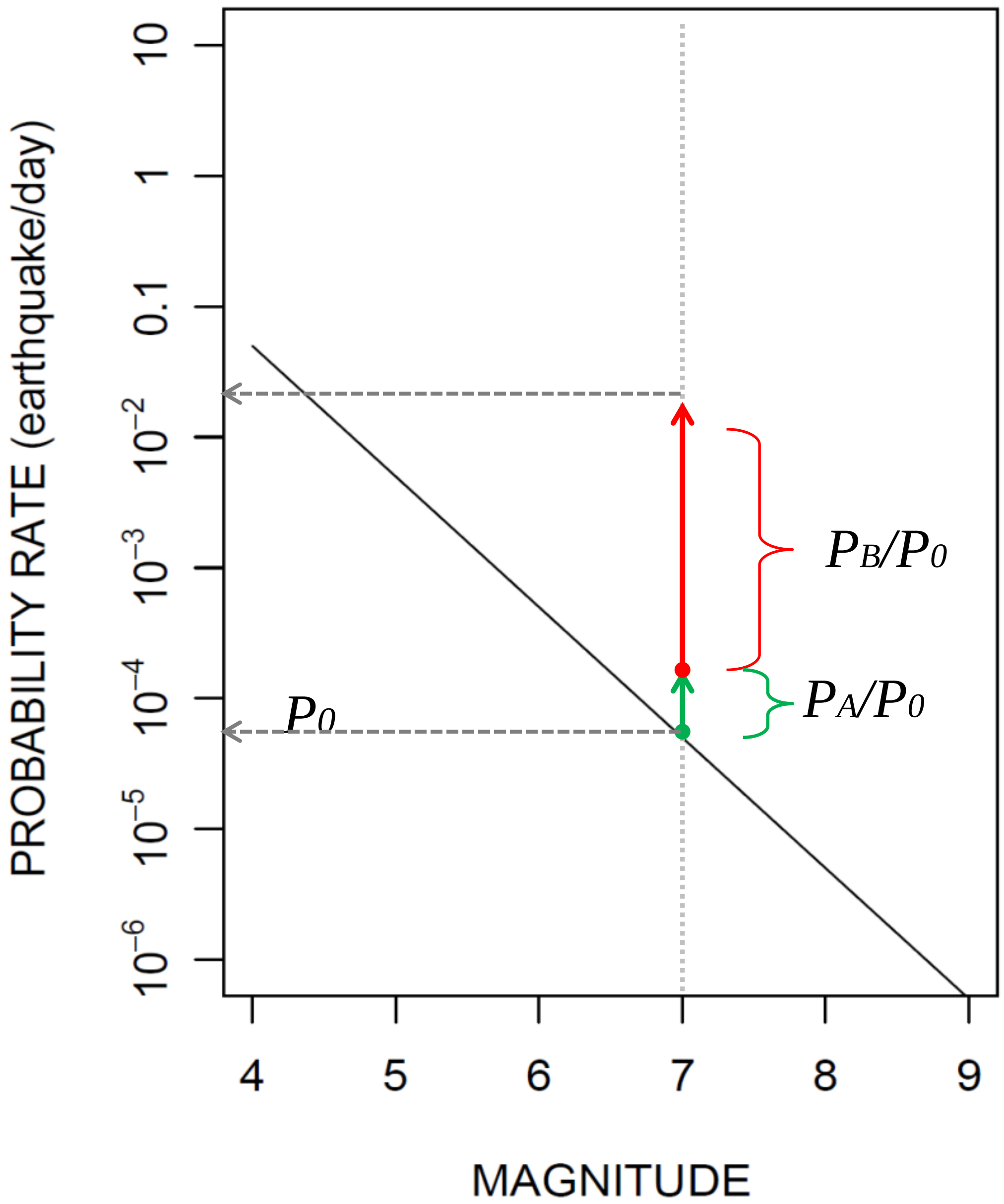


Earthquake Research Committee (2013)

Long-term probabilities of $M \geqq 7$ earthquake in Kumamoto region

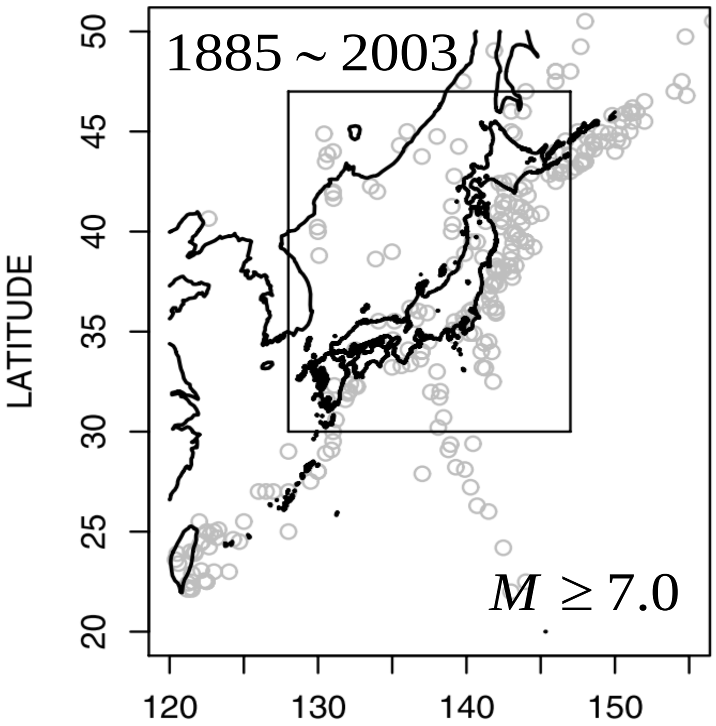
Unit time		30yrs	1yr	1mo.	1day
Secular: S1	ETAS-μ & G-R				0.00027 %
S2	#(M≧4) & G-R				0.00058 %
Long-term: A1	**Futagawa Fault**	**0.9%**			**0.000082 %**
A2	**Kumamoto Dist.**	**10.5 %**			**0.00096 %**
A3	**Central Kyushu**	**21.0 %**			**0.0019 %**
Medium-term: B1	Triggering		0.5%		0.0014 %
B2	Quiescence		2.0%		0.0055 %
Short-term: C	Foreshocks			5.0%	0.17 %

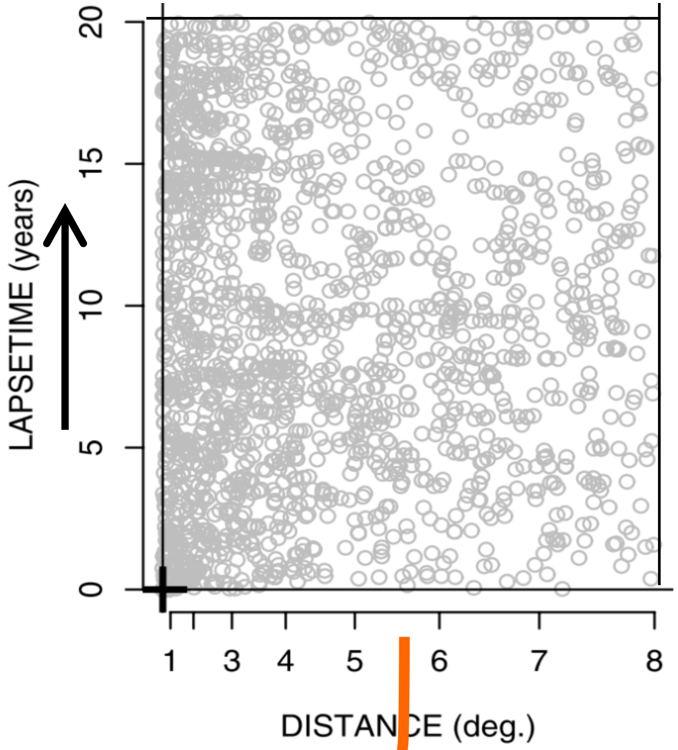
Intermediate term probability

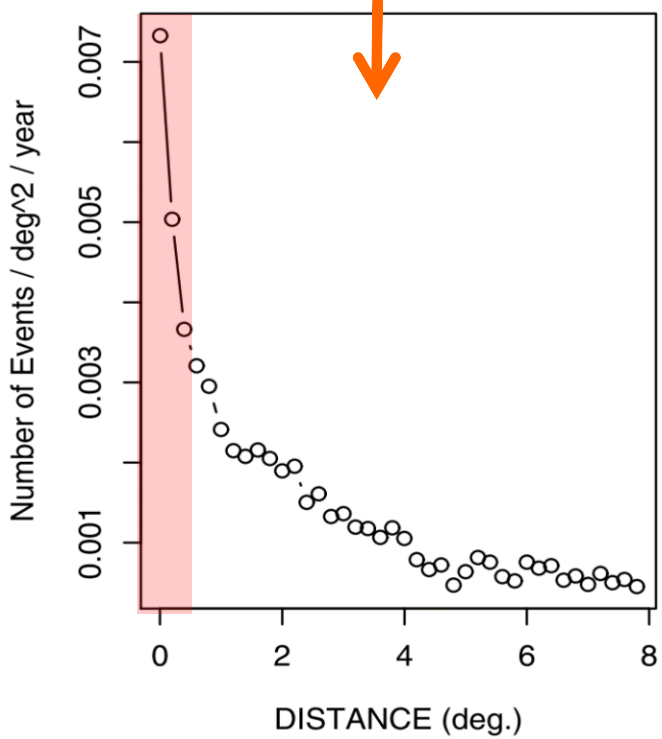


Space-time stacked pattern of earthquake triggering effect

(*RCCEP*, 2004)



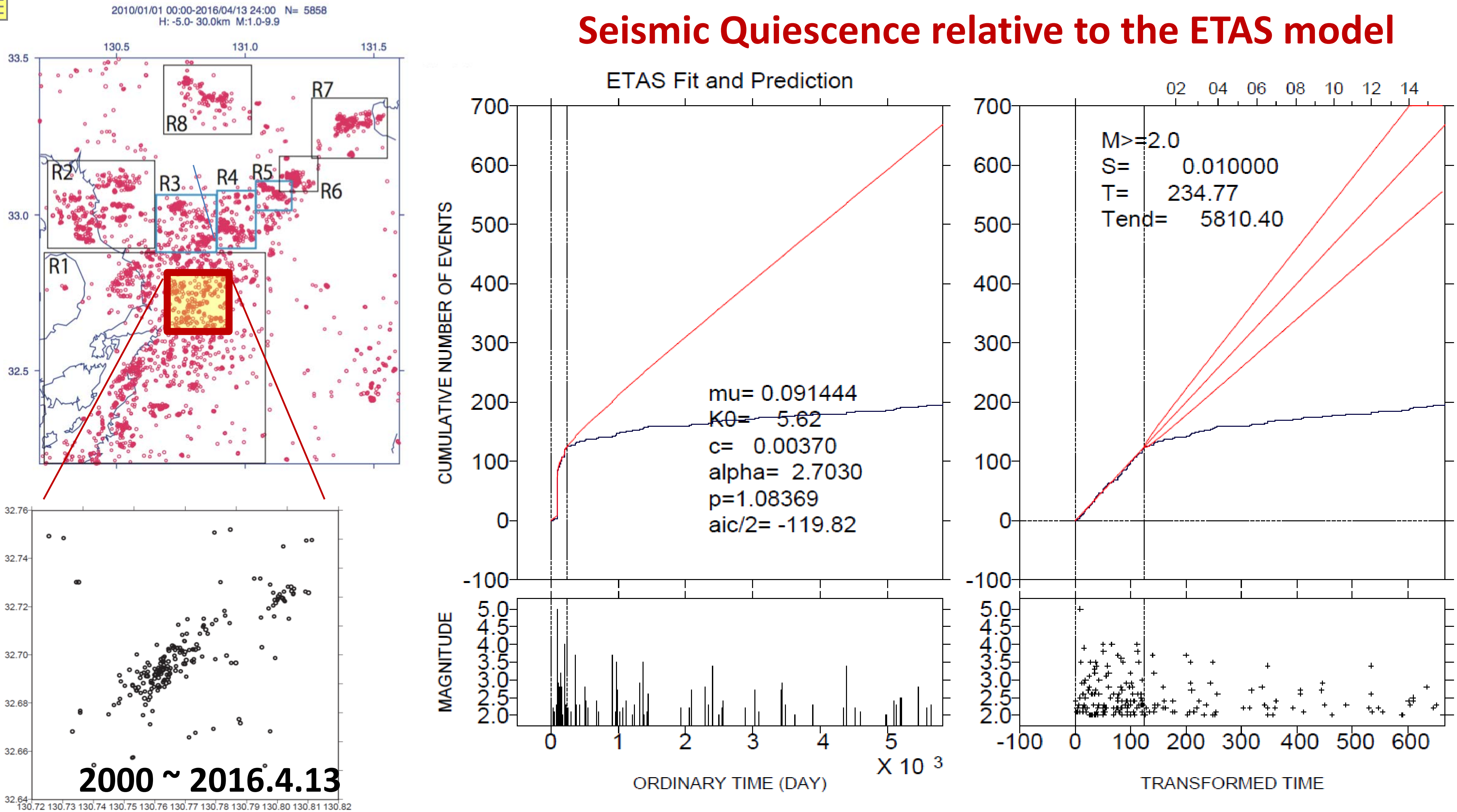




Intermediate-term probabilities of $M \geqq 7$ earthquake in Kumamoto region

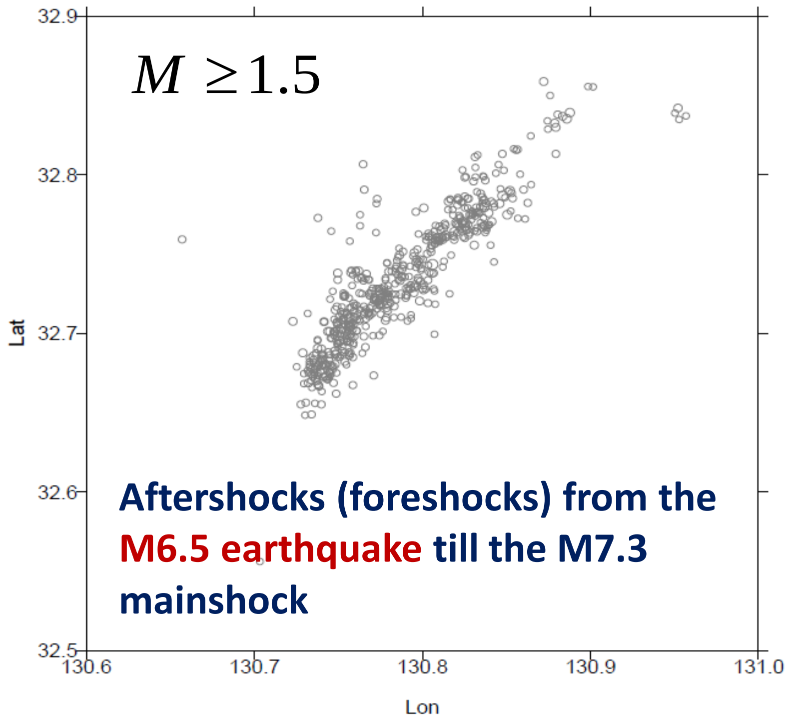
Unit time		30yrs	1yr	1mo.	1day
Secular S1	ETAS-μ & G-R				0.00027 %
S2	#(M≧4) & G-R				0.00058 %
Long-term A1	Futagawa Fault	0.9%			0.000082 %
A2	Kumamoto Dist.	10.5 %			0.00096 %
A3	Central Kyushu	21.0 %			0.0019 %
Medium-term B1	**Triggering**		**0.5%**		**0.0014 %**
B2	**Quiescence**		**2.0%**		**0.0055 %**
Short-term C	Foreshocks			5.0%	0.17 %

Seismic Quiescence relative to the ETAS model

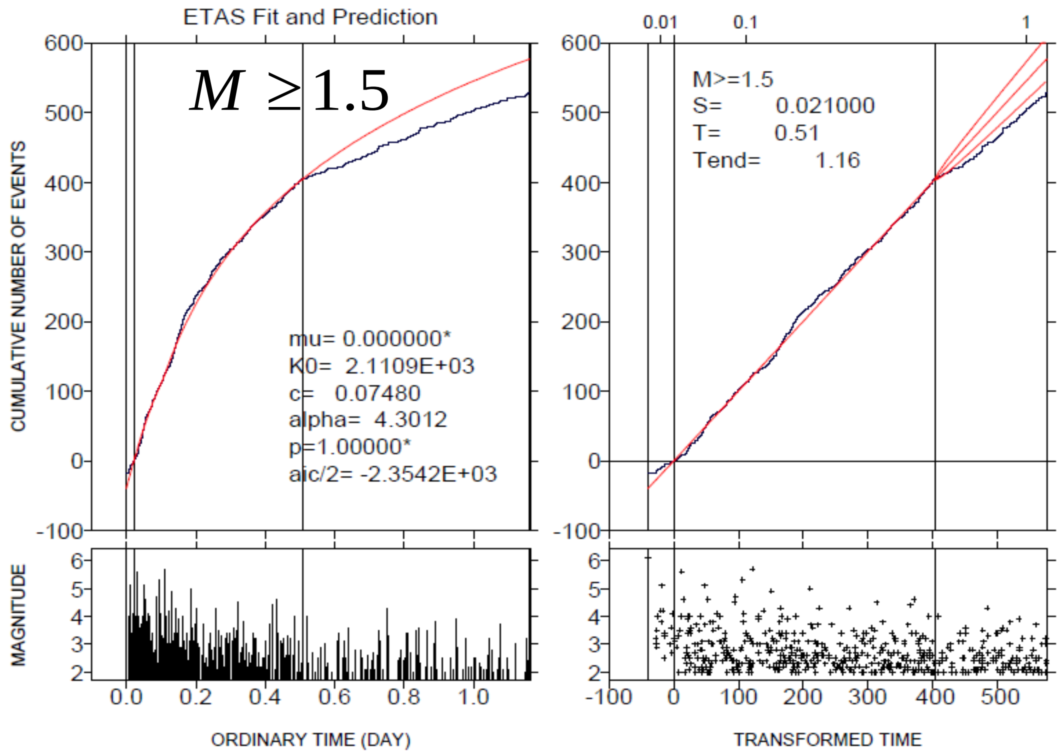


Aftershock activity of the **2000 M5.0 earthquake** that occurred in the same region as the M6.5 foreshock area became quiet relative to the ETAS model, after 4 months from the mainshock.

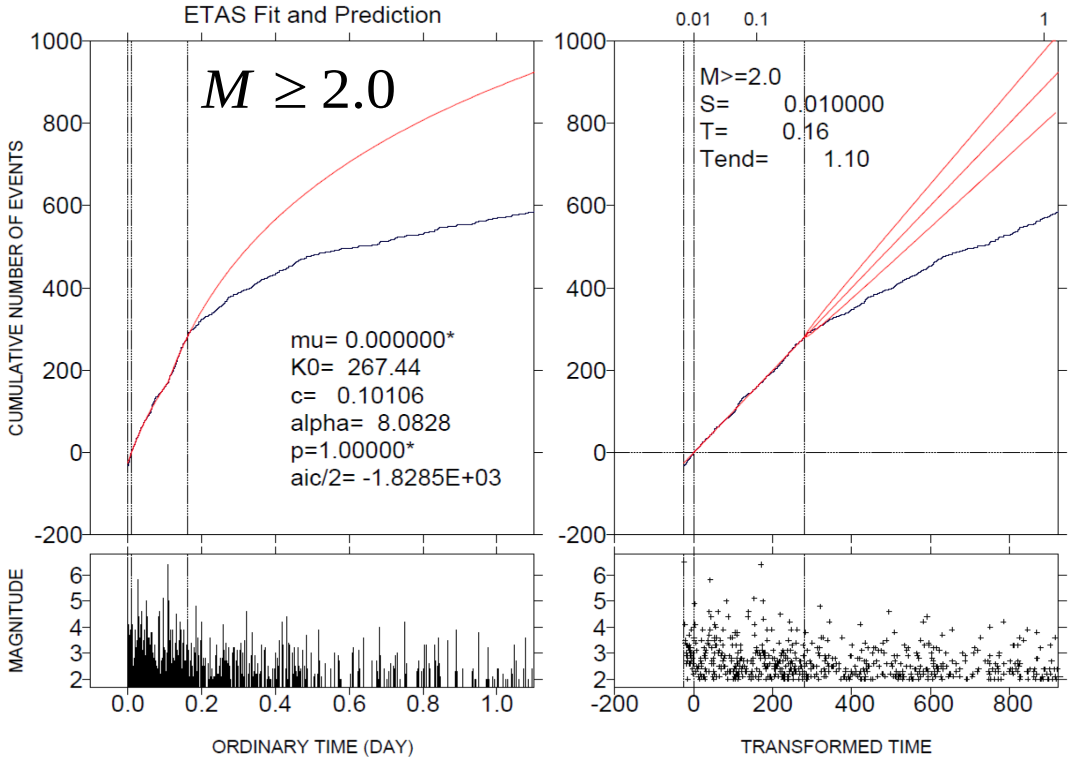
2016.4.14 ~ 4.15



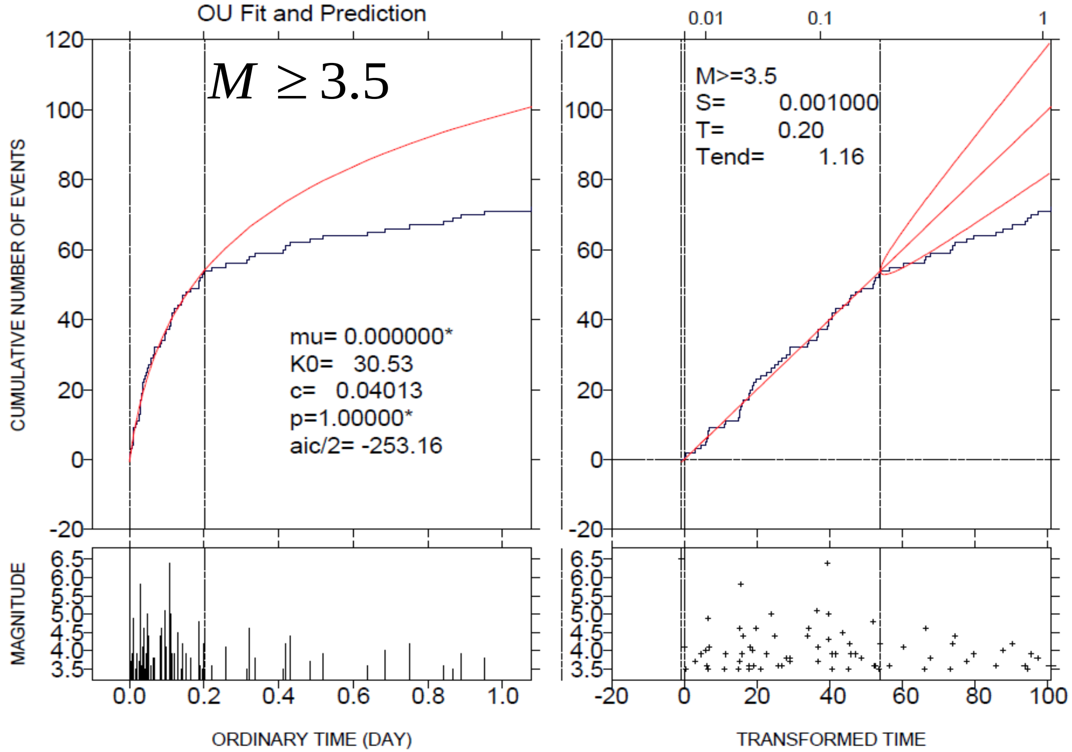
M ≥1.5
Aftershocks (foreshocks) from the M6.5 earthquake till the M7.3 mainshock
Lat
Lon
32.9
32.8
32.7
32.6
32.5
130.6
130.7
130.8
130.9
131.0



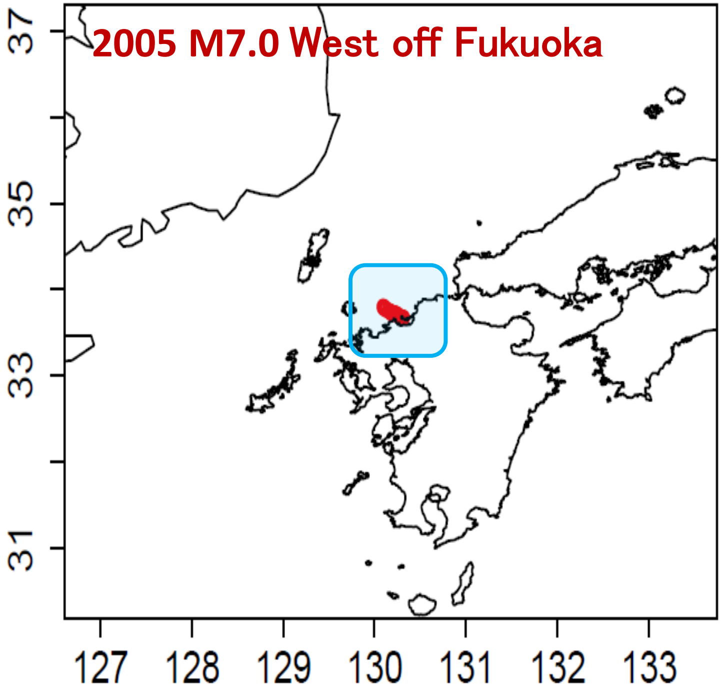
ETAS Fit and Prediction
M ≥1.5
mu= 0.000000*
K0= 2.1109E+03
c= 0.07480
alpha= 4.3012
p=1.00000*
aic/2= -2.3542E+03
M>=1.5
S= 0.021000
T= 0.51
Tend= 1.16
CUMULATIVE NUMBER OF EVENTS
MAGNITUDE
ORDINARY TIME (DAY)
TRANSFORMED TIME

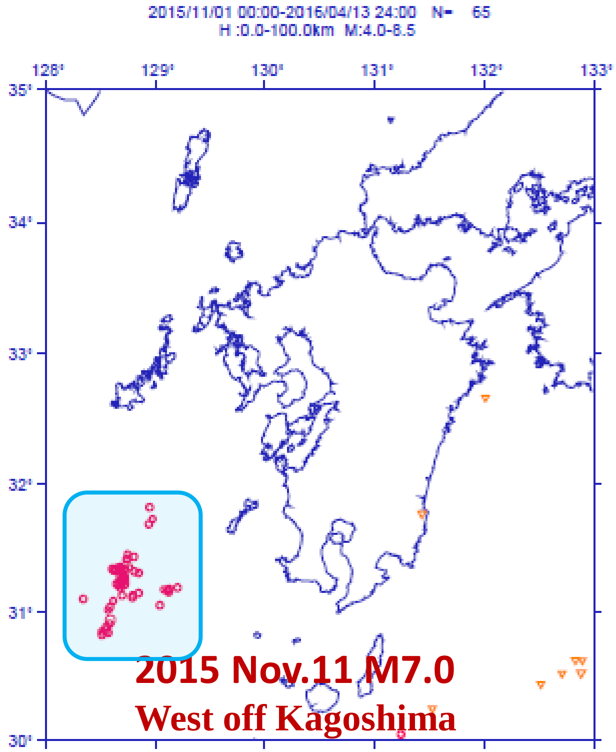


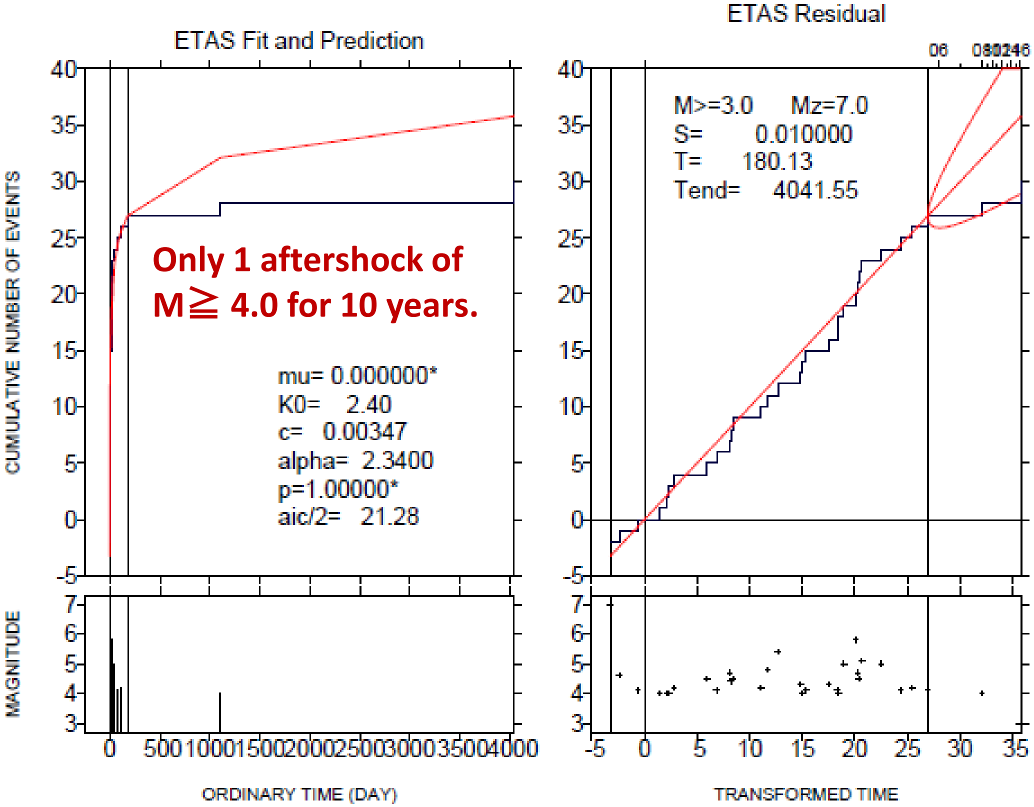
ETAS Fit and Prediction
M ≥ 2.0
mu= 0.000000*
K0= 267.44
c= 0.10106
alpha= 8.0828
p=1.00000*
aic/2= -1.8285E+03
M>=2.0
S= 0.010000
T= 0.16
Tend= 1.10
CUMULATIVE NUMBER OF EVENTS
MAGNITUDE
ORDINARY TIME (DAY)
TRANSFORMED TIME



OU Fit and Prediction
M ≥ 3.5
mu= 0.000000*
K0= 30.53
c= 0.04013
p=1.00000*
aic/2= -253.16
M>=3.5
S= 0.001000
T= 0.20
Tend= 1.16
CUMULATIVE NUMBER OF EVENTS
MAGNITUDE
ORDINARY TIME (DAY)
TRANSFORMED TIME


2005 M7.0 West off Fukuoka
37
35
33
31
127
128
129
130
131
132
133


2015/11/01 00:00-2016/04/13 24:00 N= 65
H :0.0-100.0km M:4.0-8.5
2015 Nov.11 M7.0
West off Kagoshima


ETAS Fit and Prediction
ETAS Residual
Only 1 aftershock of
M≧ 4.0 for 10 years.
mu= 0.000000*
K0= 2.40
c= 0.00347
alpha= 2.3400
p=1.00000*
aic/2= 21.28
M>=3.0 Mz=7.0
S= 0.010000
T= 180.13
Tend= 4041.55
CUMULATIVE NUMBER OF EVENTS
MAGNITUDE
ORDINARY TIME (DAY)
TRANSFORMED TIME

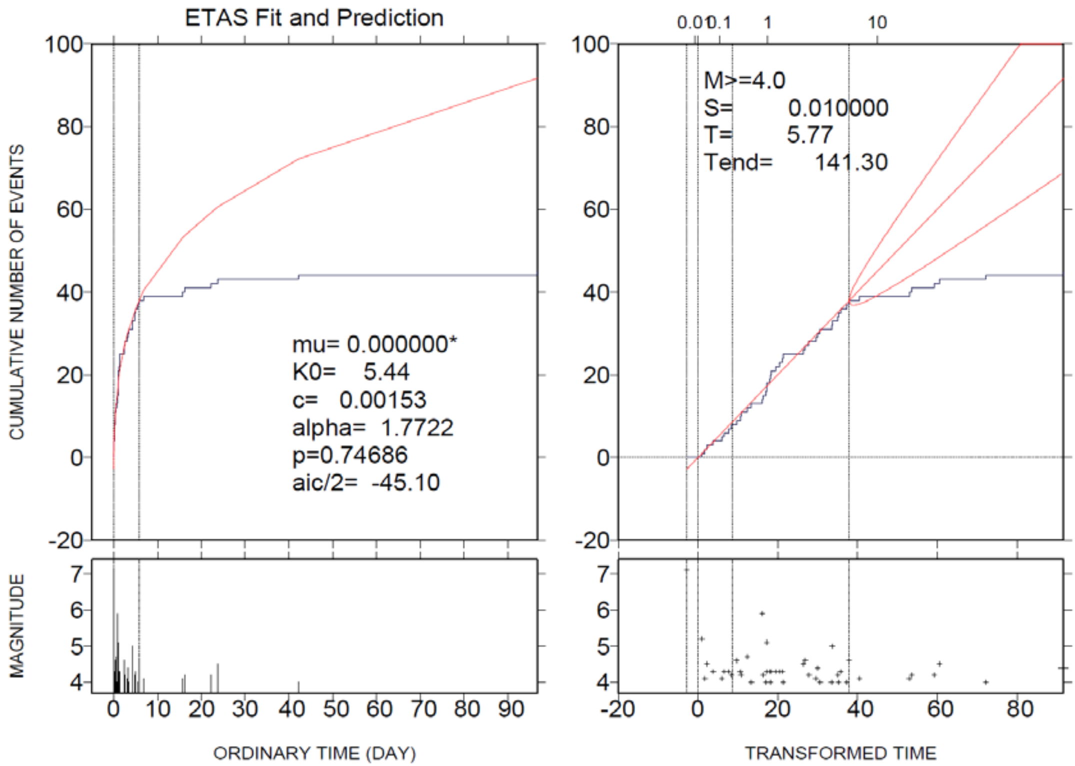
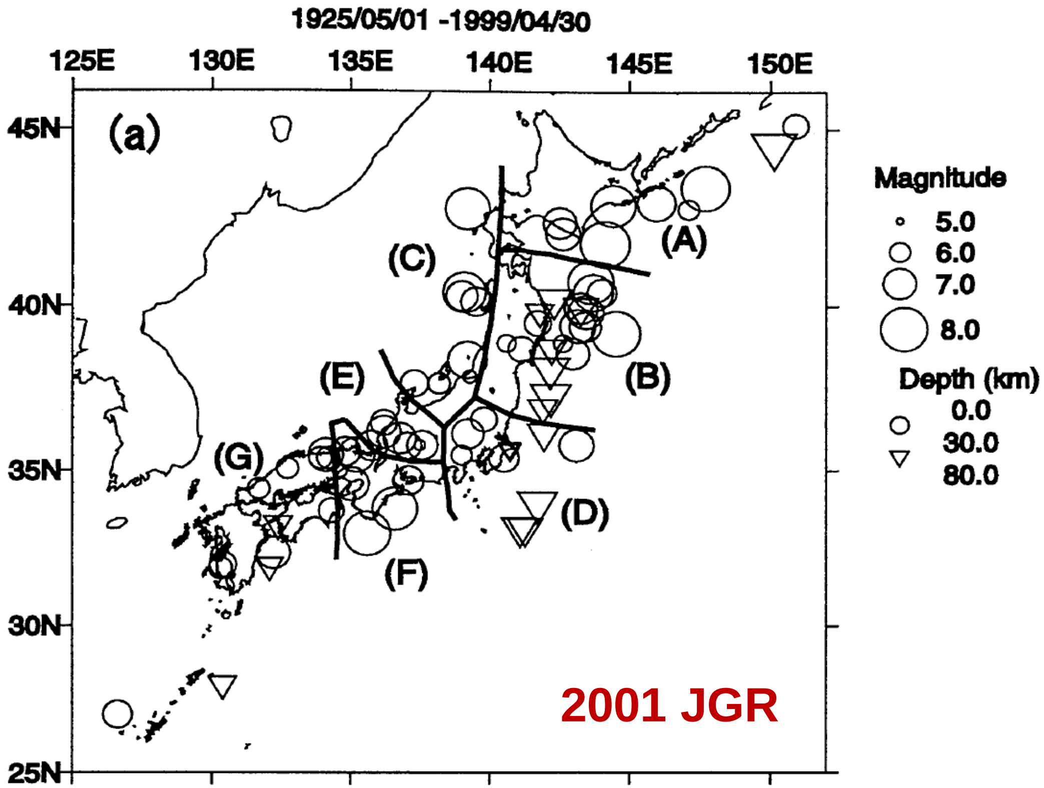

ETAS Fit and Prediction
mu= 0.000000*
K0= 5.44
c= 0.00153
alpha= 1.7722
p=0.74686
aic/2= -45.10
M>=4.0
S= 0.010000
T= 5.77
Tend= 141.30
CUMULATIVE NUMBER OF EVENTS
MAGNITUDE
ORDINARY TIME (DAY)
TRANSFORMED TIME

Table 1. Studied Aftershocks[a]

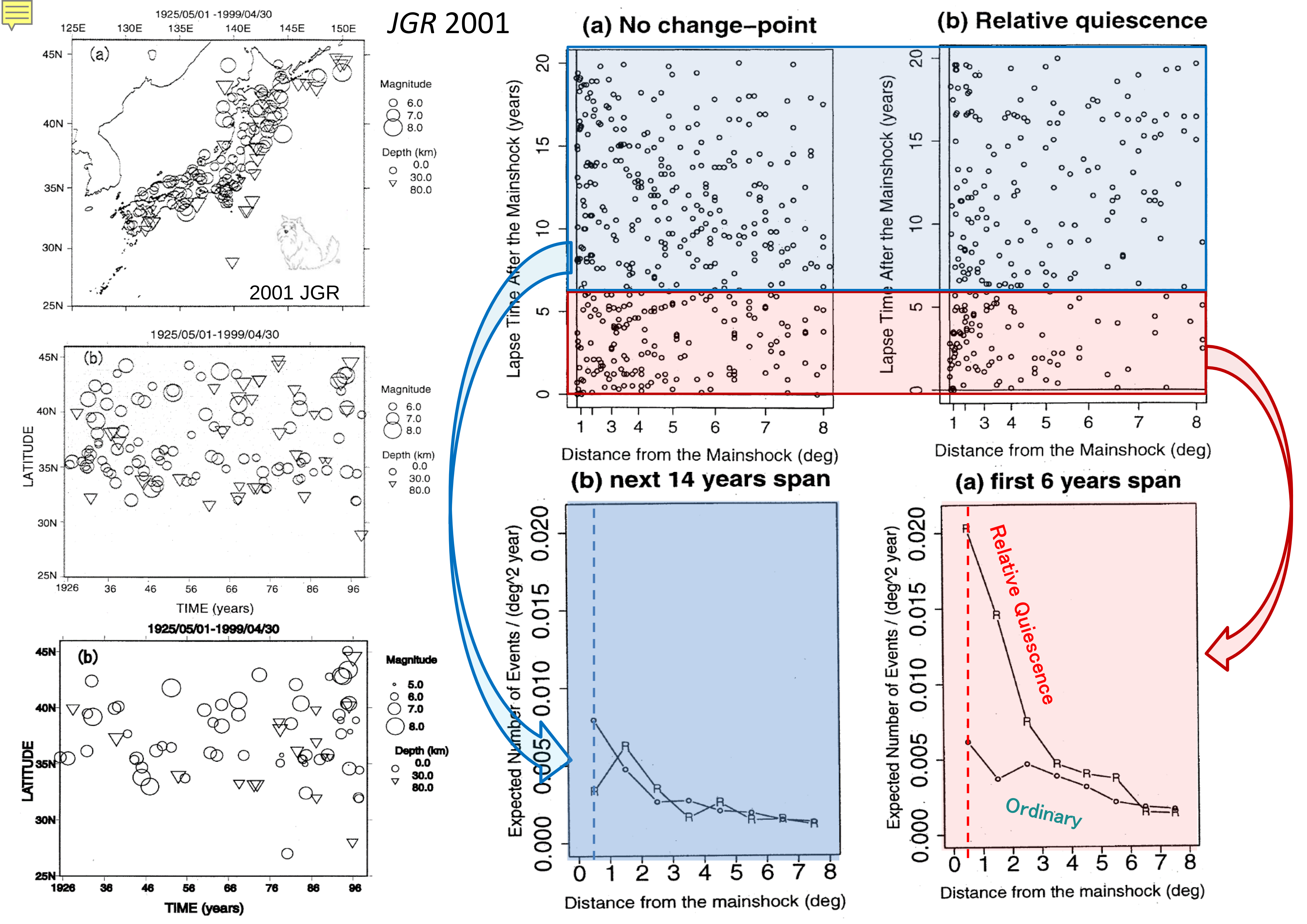
Date	Name	M_J	Epicenter	Threshold Magnitudes[b]
Off the East Coast of Hokkaido (Region A)				
Dec. 04, 1995	Off Iturup Island	7.9	150.1 44.6	(4.0) (4.5) 5.0
Oct. 04, 1994	Hokkaido-Toho-Oki	8.1	147.7 43.4	4.5 5.0 (5.5*A*) 6.0
Aug. 18, 1994	Hokkaido-Toho-Oki	6.4	150.9 45.1	(3.5) (4.0) (4.5)
Jan. 15, 1993	Kushiro-Oki	7.8	144.4 42.9	3.0 3.5 4.0
April 01, 1990	Hokkaido-Toho-Oki	6.0	147.1 42.8	(0.0) (3.6) (3.8) (4.0)
March 21, 1982	Urakawa-Oki	7.1	142.6 42.1	3.3 3.6 4.0 4.2
June 17, 1973	Nemuro-Hanto-Oki	7.4	146.0 43.0	(4.0) (4.5) (4.8) 5.2
March 04, 1952	Tokachi-Oki	8.2	144.1 41.8	0.0 5.0 5.5 6.0
Nov. 26, 1932	Hidaka-Chubu	7.0	142.5 42.4	unfelt (felt) (4.5)
Off the East Coast and Inland of Tohoku District (Region B)				
Aug. 11, 1996	Onikobe	5.9	140.6 38.9	2.5 3.1 3.5 (3.8) (4.0)
Jan. 07, 1995	Sanriku-Haruka-Oki (secondary)	7.2	142.3 40.2	3.0 3.5 4.0 4.5
Dec. 28, 1994	Sanriku-Haruka-Oki (long)	7.5	143.7 40.4	4.0 4.5 5.0 5.4
Dec. 28, 1994	Sanriku-Haruka-Oki (short)	7.5	143.7 40.4	4.0 (4.5) (4.9)
April 08, 1994	Sanriku-Oki	6.6	144.0 40.6	3.5 4.0
Dec 28, 1992	Sanriku-Oki	5.9	142.6 38.9	(3.0) (3.5) (3.9) (4.4)
July 18, 1992	Sanriku-Haruka-Oki	6.9	143.7 39.4	(3.5) (3.7) 3.8 4.0 (4.5) 5.0
Nov. 02, 1989	Iwate-ken-Oki	7.1	143.1 35.8	4.0 4.5 5.0
Jan. 09, 1987	Iwate-Ken-Hokubu	6.6	141.8 39.8	0.0 3.0
Jan. 19, 1981	Miyagi-Ken-Oki	7.0	143.0 38.6	0.0 3.6 4.1 4.6
June 12, 1978	Miyagi-Ken-Oki	7.4	142.2 38.2	3.4 4.0 4.2 (4.5) (4.9)
Feb. 20, 1978	Near Ojika-Peninsula	6.7	142.2 38.8	3.0 3.3 (3.5)
June 12, 1968	Tokachi-Oki (Southern)	7.2	143.1 39.4	4.5 5.0 5.5
May 16, 1968	Tokachi-Oki (Northern)	7.9	143.6 40.7	4.5 5.0 5.5 5.9
April 30, 1962	Miyagi-Ken-Hokubu	6.5	141.1 38.7	(0.0) 4.0
March 21, 1960	Iwate-Ken-Oki	7.2	143.4 39.8	0.0 4.4 4.9 5.4
Nov. 05, 1938	Shioya-Oki-Swarm	7.5	142.2 37.3	4.5 4.7 (5.0) 5.5
March 03, 1933	Sanriku-Oki	8.1	144.5 39.2	(0.0*A*)(6.0*N*)5.5(5.8*A*)(6.0)(6.2)
Nov. 04, 1931	Iwate-Ken-Tobu	6.5	141.7 39.5	(unfelt) (felt) (felt*A*) 4.0
May 27, 1928	Iwate-Ken-Oki	7.0	143.3 40.0	0.0 5.1 (5.3) (5.5)
Eastern Limb in Sea of Japan (Region C)				
Apr. 01, 1995	Niigata-Ken-Chubu	5.5	139.3 37.9	(2.8*A*) 3.0 3.2 (3.5)
July 12, 1993	Hokkaido-Nansei-Oki	7.8	139.2 42.8	(4.0*A*) 4.5 5.0
Feb. 07, 1993	Noto-Hanto-Oki	6.6	137.3 37.7	(3.0) 3.5 (4.0)
May 26, 1983	Nihonkai-Chubu	7.7	139.1 40.4	4.0 4.5 (5.0) (5.2) (5.3)
June 16, 1964	Niigata earthquake	7.5	139.2 38.4	felt 4.0 4.5 5.0
May 07, 1964	Oga-Hanto-Oki	6.9	139.0 40.3	unfelt felt 0.0
May 01, 1939	Oga-Hanto-Oki	6.8	139.5 40.1	0.0 4.1
Kanto and Tokai District and Their Offshore Regions (Region D)				
June 01, 1990	Chiba-Ken-Hokubu	6.0	140.7 35.7	2.0 2.5 3.0
March 06, 1989	Chiba-Ken-Hokubu	6.0	140.7 35.6	(2.0) (2.5) 3.0 (3.5)
Dec. 17, 1987	Chiba-Ken-Toho-Oki	6.7	140.5 35.4	(2.5) (3.0) (3.5) (4.0)
Aug 08, 1983	Yamanashi-Ken-Tobu	6.0	139.0 35.5	2.5 (2.7) (2.9) (3.0)
July 23, 1982	Ibaragi-Ken-Oki	7.0	142.0 36.2	3.2 (3.7) 4.2
Dec. 04, 1972	Hachijo-Jima-Oki	7.2	141.1 33.2	3.6 4.0 4.5
Feb. 29, 1972	Hachijo-Jima-Oki	7.1	141.3 33.2	(3.6) (4.0) (4.6)
Nov. 26, 1953	Boso-Oki	7.4	141.7 34.0	0.0 5.0 5.5
Dec. 26, 1949	Imaichi	6.4	139.8 36.6	felt 4.2
May 23, 1938	Ibaragi-Ken-Oki	7.3	141.6 36.7	0.0 4.1
Sept. 21, 1931	Saitama-Ken-Seibu	6.9	139.2 36.2	0.0 3.0 3.5 (3.8) (4.0)
Hokuriku and Chubu District (Region E)				
Sept. 14, 1984	Nagano-Ken-Seibu	6.8	137.6 35.8	3.5 4.0 4.5
Oct. 07, 1978	Nagano-Ken-Swarm	5.3	137.5 35.8	(2.5) (3.0) 3.5
Sept. 09, 1969	Gifu-Ken-Chubu	6.6	137.1 35.8	0.0 3.9
March 27, 1963	Echizen-Misaki-Oki	6.9	135.8 35.8	0.0
Aug. 19, 1961	Kita-Mino	7.0	136.8 36.0	0.0
March 07, 1952	Daishoji-Oki	6.5	136.2 36.5	(felt) (0.0) (4.2) (4.5)
June 28, 1948	Fukui	7.1	136.2 36.2	felt 0.0 3.5 4.0 (4.5) 4.7
July 15, 1941	Nagano	6.1	138.2 36.7	(felt) (0.0)

Table 1. (continued)

Date	Name	M_J	Epicenter	Threshold Magnitudes[b]
Kinki District and Offshore Regions (Region F)				
Jan. 17, 1995	Hyogo-Ken-Nanbu	7.2	135.0 34.6	3.0 3.5 4.0 4.2
Nov. 09, 1994	Inagawa Swarm	4.0	135.4 34.9	(2.0) (2.3) (2.5) (2.6)
May 30, 1984	Yamasaki Fault	5.6	134.6 35.0	(2.5) 2.6 3.0
Dec. 21, 1946	Nankaido	8.0	135.6 33.0	(4.0) (4.5) (4.9) 5.0 5.5
Jan. 13, 1945	Mikawa	6.8	137.1 34.7	(felt) (0.0) (4.4) 4.8
Dec. 07, 1944	Tonankai	7.9	136.6 33.8	4.0 (4.5) (4.8) 5.0
March 07, 1927	Kita-Tango	7.3	135.2 35.5	0.0 4.5
May 23, 1925	Tajima	6.8	134.8 35.6	(felt) (5.0)
Southwestern Japan (Region G)				
June 25, 1997	Yamaguchi/Shimane-Ken Border	6.1	131.7 34.5	(2.6) (3.0) 3.4
May 13, 1997	Northern Satsuma	6.2	130.3 31.9	(2.5) 2.8 3.0 3.3
March 26, 1997	Northern Satsuma	6.5	130.4 32.0	(2.7) (3.0) (3.5)
Oct. 18, 1995	Amami-Oshima-Oki	6.6	130.4 28.0	(3.5) 3.8 4.0 4.5
March 18, 1987	Miyazaki-Ken-Oki	6.6	132.1 32.0	2.5 2.9 3.4
Aug. 07, 1984	Miyazaki-Ken-Oki	7.1	132.2 32.4	2.8 3.3 3.8
Oct 31, 1983	Tottori-Ken	6.2	133.9 35.4	(2.3) (2.5) (2.7) (2.8) 3.0
Mar 03, 1980	Okinawa-Hokusei-Oki	6.7	126.6 27.0	(0.0) (4.2) (4.5)
June 04, 1978	Shimane-Ken-Chubu	6.1	132.7 35.1	0.0 3.3 3.7
Aug. 06, 1968	Ehime-Ken-Seigan	6.6	132.4 33.3	3.5 4.0
July 27, 1955	Tokushima-Ken-Nanbu	6.4	134.3 33.8	0.0 3.0
Sept. 10, 1943	Tottori	7.4	134.1 35.5	(4.0) (4.4) (4.7) 5.0
March 04, 1943	Eastern Tottori	6.2	134.2 35.4	(felt) (3.6)



JGR 2001

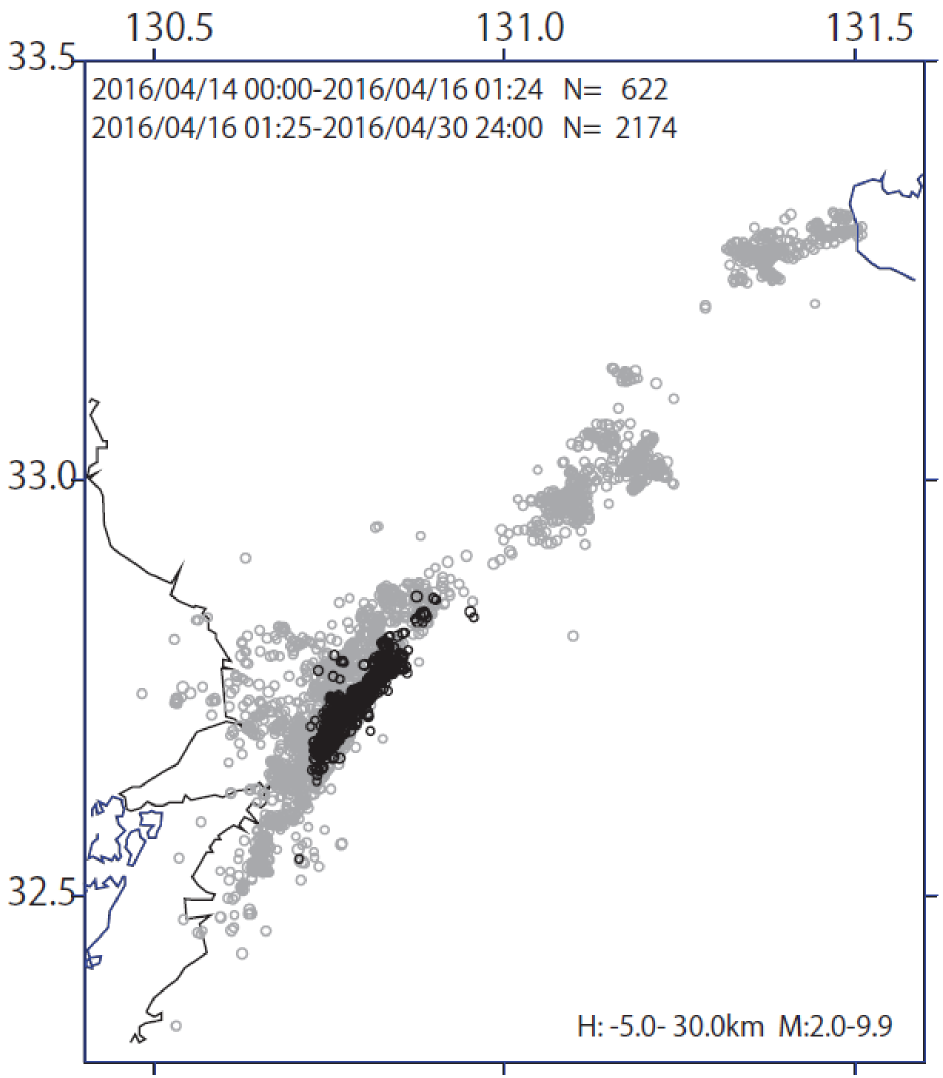


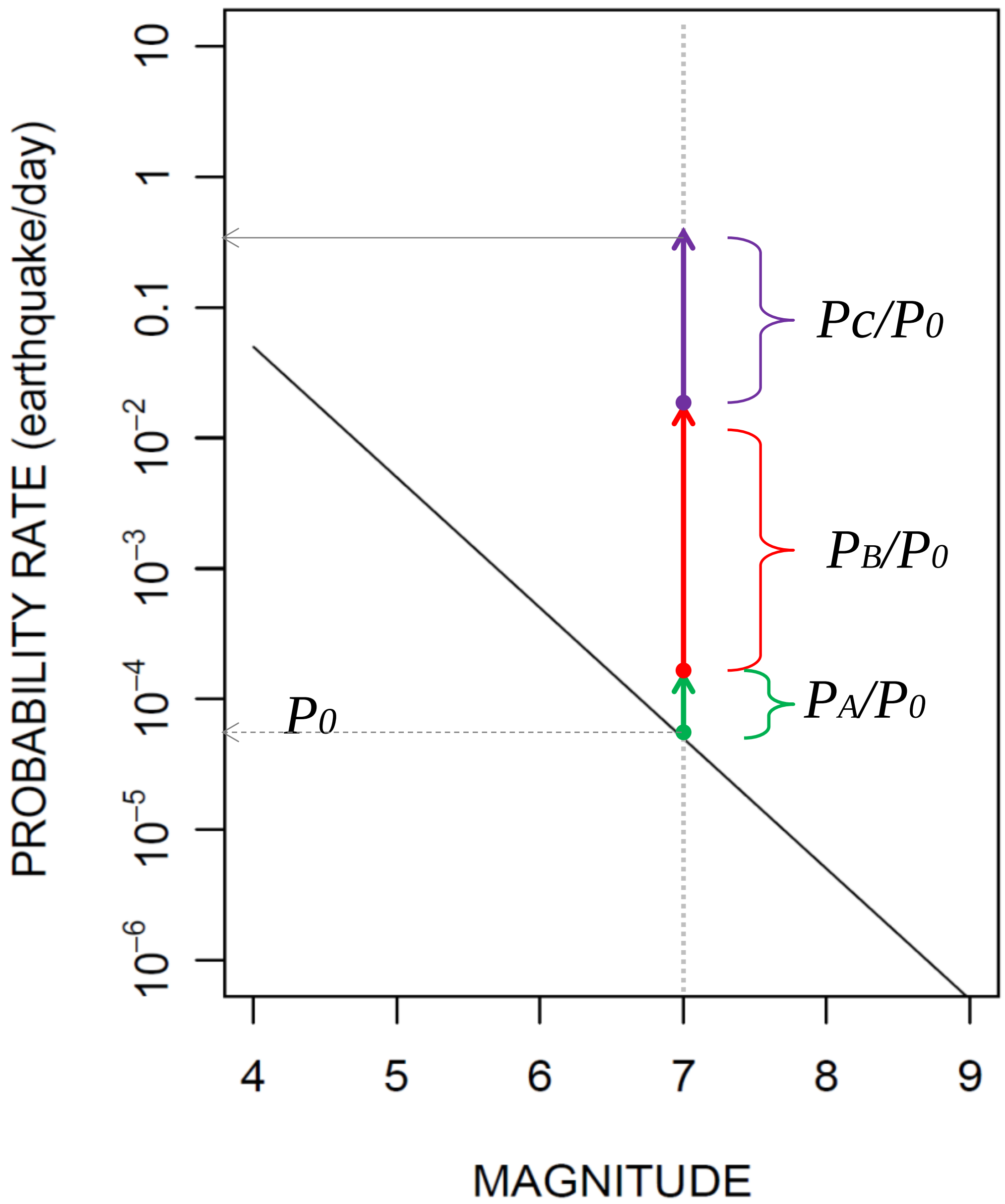
1925/05/01 -1999/04/30
(a)
2001 JGR
Magnitude
6.0
7.0
8.0
Depth (km)
0.0
30.0
80.0
1925/05/01-1999/04/30
(b)
LATITUDE
TIME (years)
1925/05/01-1999/04/30
(b)
Magnitude
5.0
6.0
7.0
8.0
Depth (km)
0.0
30.0
80.0
LATITUDE
TIME (years)
(a) No change-point
(b) Relative quiescence
Lapse Time After the Mainshock (years)
Distance from the Mainshock (deg)
(b) next 14 years span
(a) first 6 years span
Expected Number of Events / (deg^2 year)
Distance from the mainshock (deg)
Relative Quiescence
Ordinary

Intermediate-term probabilities of $M \geqq 7$ earthquake in Kumamoto region

Unit time		30yrs	1yr	1mo.	1day
Secular S1	ETAS-μ & G-R				0.00027 %
S2	#(M≧4) & G-R				0.00058 %
Long-term A1	Futagawa Fault	0.9%			0.000082 %
A2	Kumamoto Dist.	10.5 %			0.00096 %
A3	Central Kyushu	21.0 %			0.0019 %
Medium-term **B1**	**Triggering**		**0.5%**		**0.0014 %**
B2	**Quiescence**		**2.0%**		**0.0055 %**
Short-term C	Foreshocks			5.0%	0.17 %

Foreshock probability prediction





Foreshock forecast (1st event)

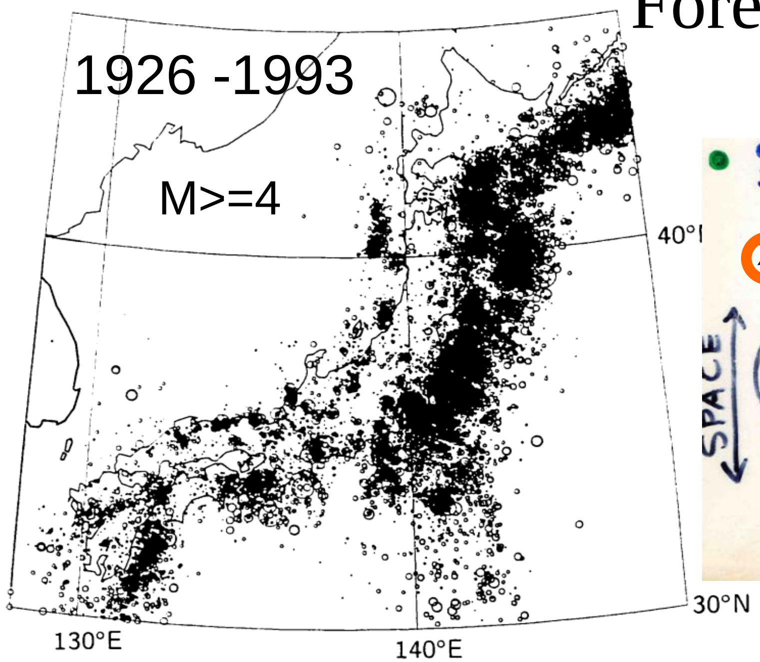
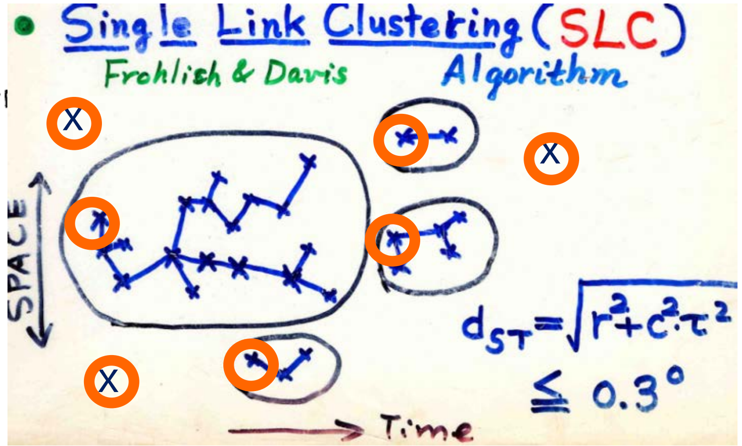
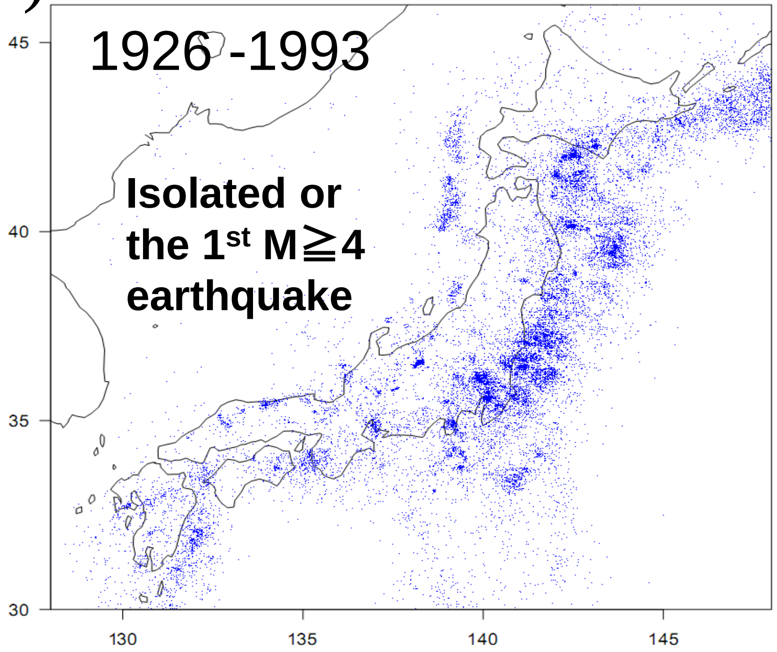
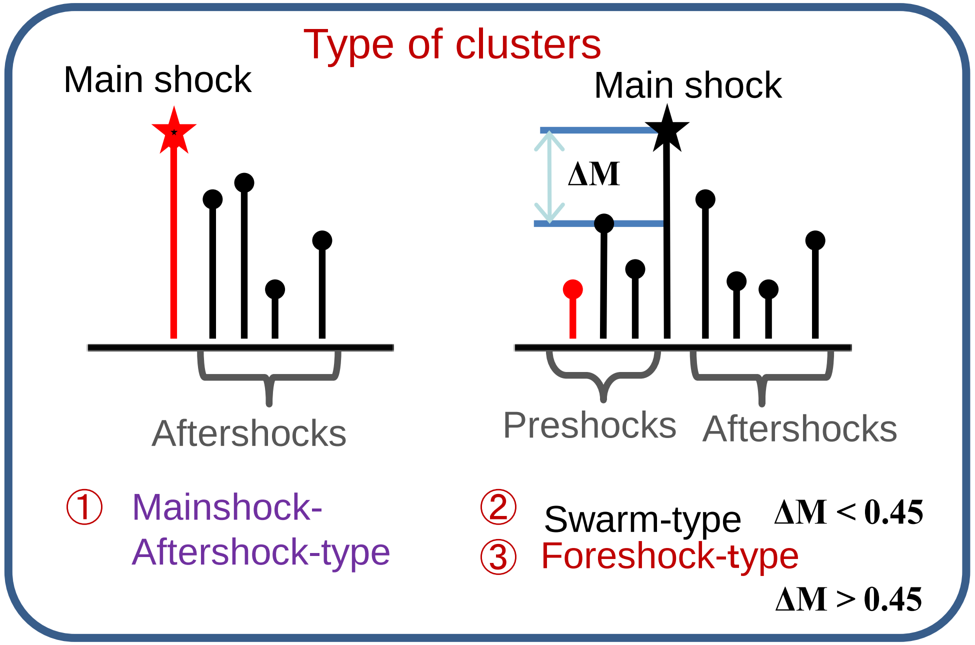


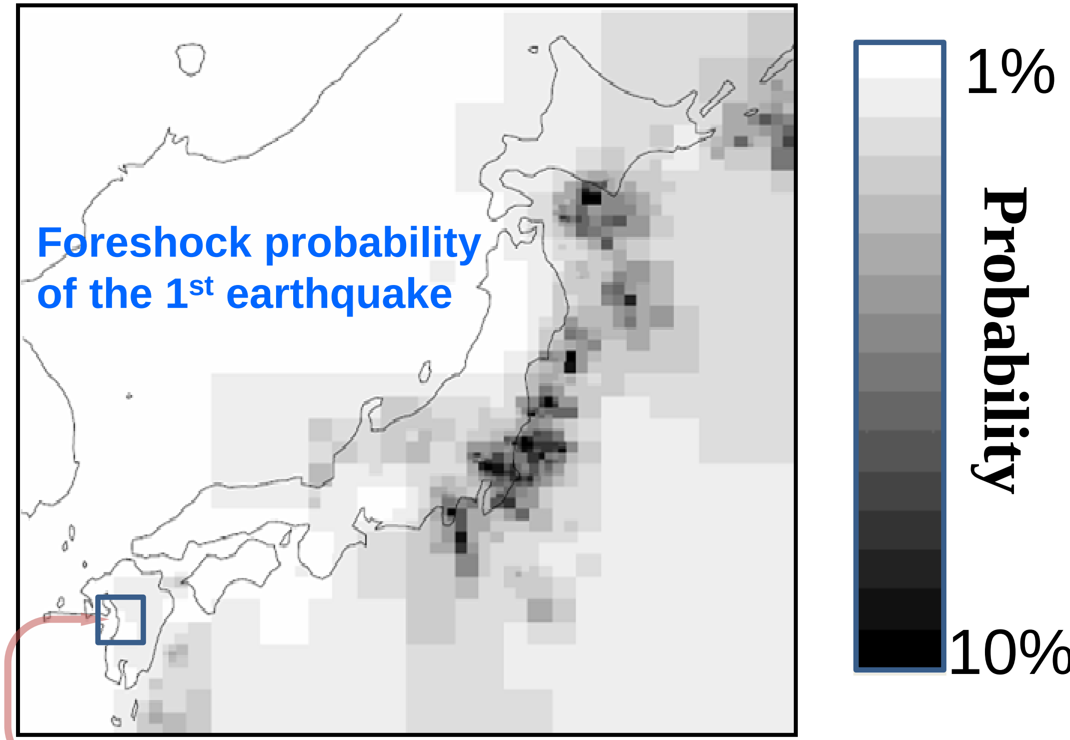
Figure 1. Epicentres of earthquakes ($M_J \geqslant 4.0$, depth $\leqslant 100$ km) in the JMA catalogue (1926–91).



$$c = 1^{o} \; / \; month \approx 1 \; km \; / \; day$$







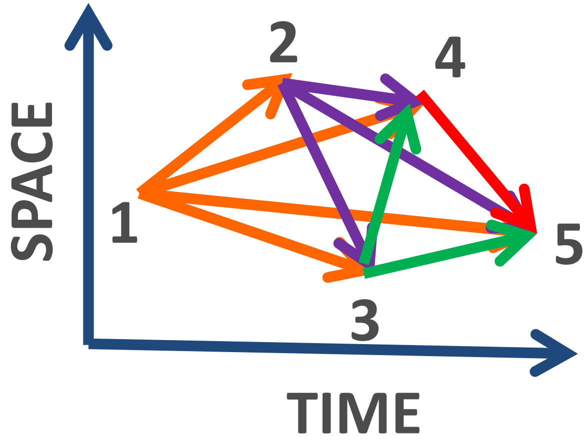
~ 2% per month for Kumamoto

Multiple earthquakes in a cluster

$$\left\{\left(t_i^c, x_i^c, y_i^c, M_i^c\right);\ i = 1, 2, \cdots, \#c\right\}$$

Hypocenters in a cluster $c \in$ all SLC clusters

Inter-events concentrations in a cluster *c* and magnitude increments



Time difference:

$$t_{i,j}^c = t_j^c - t_i^c$$

Epicenter separation:

$$r_{i,j}^c = \sqrt{(x_i^c - x_j^c)^2 + (y_i^c - y_j^c)^2}$$

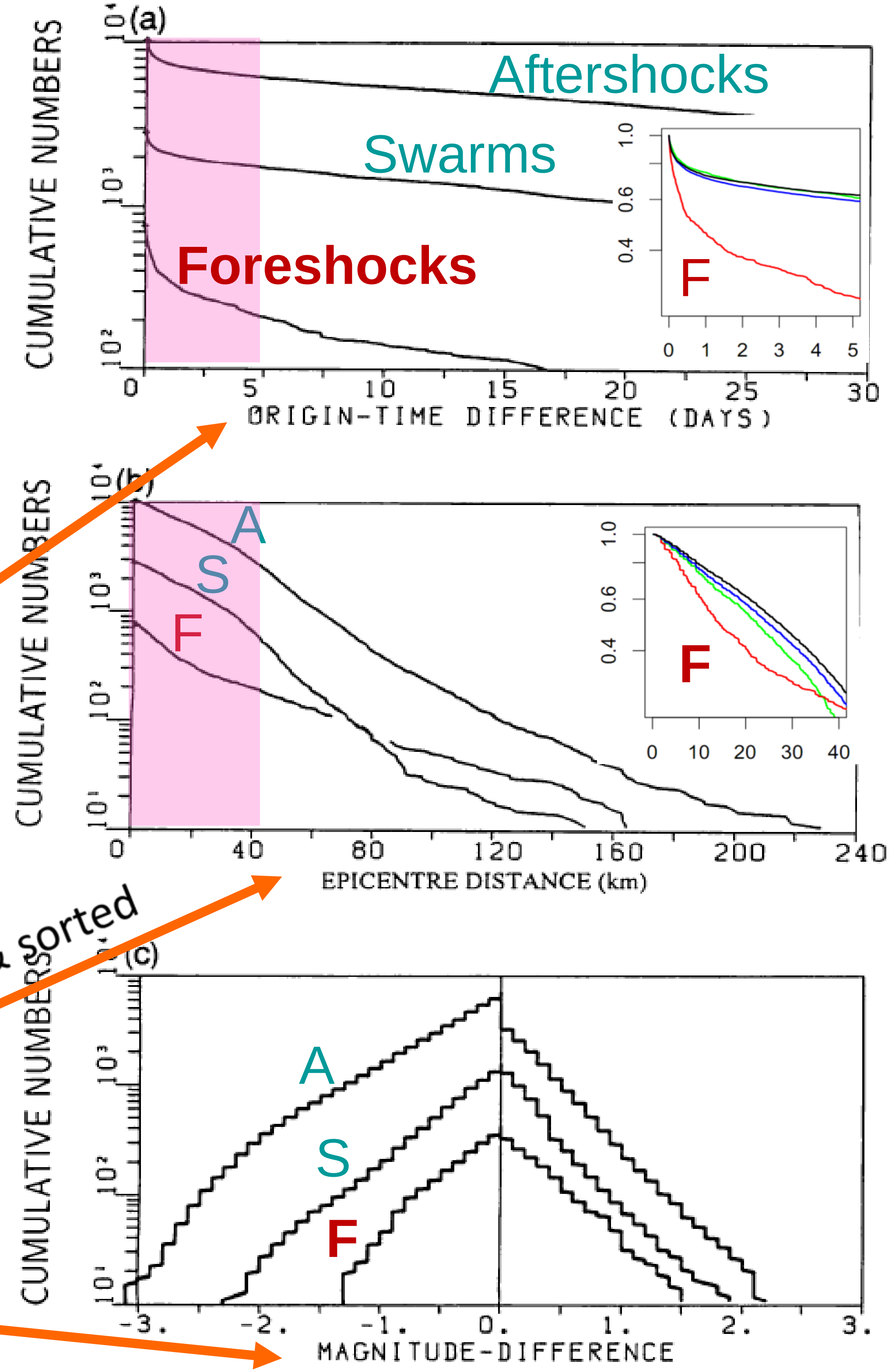
Magnitude difference:

$$g_{i,j}^c = M_j^c - M_i^c$$

Stacked & sorted

Stacked & sorted

Stacked & sorted



Ogata, Utsu & Katsura, *GJI*, 1996

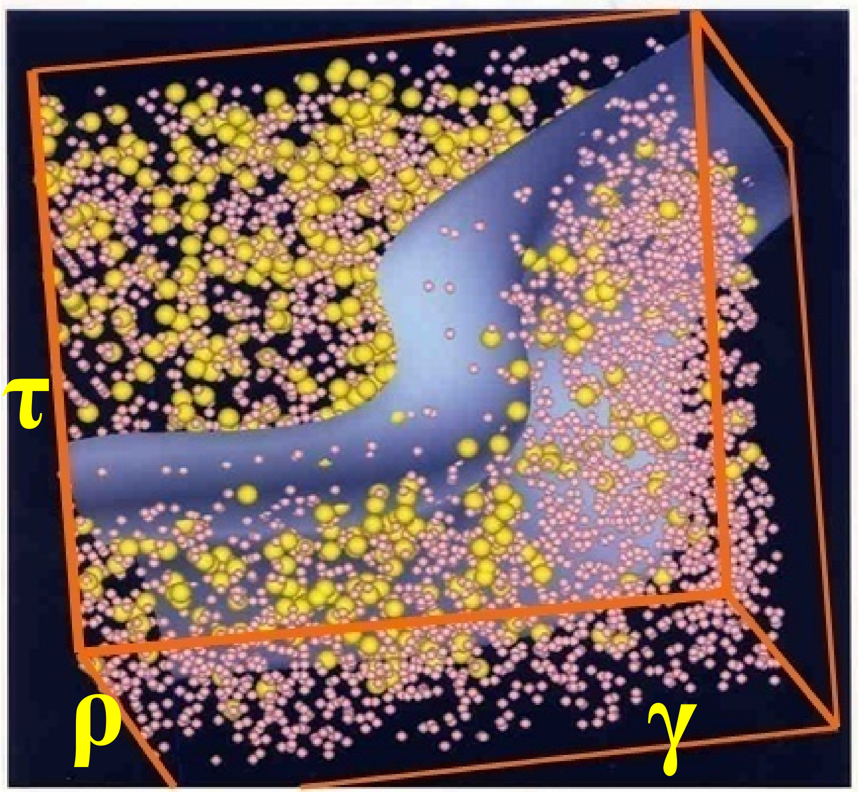
Time difference: $t^c_{i,j} = t^c_j - t^c_i$

Epicenter separation: $r^c_{i,j} = \sqrt{(x^c_i - x^c_j)^2 + (y^c_i - y^c_j)^2}$

Magnitude difference: $g^c_{i,j} = M^c_j - M^c_i$

$$(\tau^c_{i,j},\ \rho^c_{i,j},\ \gamma^c_{i,j}) \in [0,1]^3$$

unit cube



① **Time Interval Transformation:** $t \rightarrow \tau$

$$\tau = \begin{cases} 0 & \text{for } t \le 0.01 \\ \log(100t)/\log(3000) & \text{for } 0.01 < t \le 30 \\ 1 & \text{for } 30 \le t \end{cases}$$

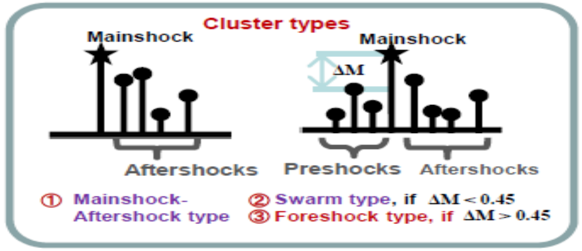
② **Epicenter Separation Transformation:** $r \rightarrow \rho$

$$\rho = 1 - \exp\{-\min(r, 50)/20\}$$

③ **Magnitude Difference Transformation:** $g \rightarrow \gamma$

$$\gamma = \begin{cases} (2/3)\exp\{g/\sigma_1\} & \text{for } g \le 0 \\ (2/3) + (1/3)[1 - \exp\{-g/\sigma_2\}] & \text{for } g > 0 \end{cases}$$

where $\sigma_1 = 6709, \sigma_2 = 0.4456$



$$f = \text{logit}(p) \triangleq \log\left\{\frac{p}{1-p}\right\} \Leftrightarrow p = \frac{e^f}{1+e^f}$$

$$\text{logit}(p_{\gamma,\rho,\tau}) \triangleq \log\left\{\frac{1-p_{\gamma,\rho,\tau}}{p_{\gamma,\rho,\tau}}\right\} = a + \sum_{k=1}^{3} b_k \gamma^k + \sum_{k=1}^{3} c_k \rho^k + \sum_{k=1}^{3} d_k \tau^k$$

GJI 1996 Algorithm of foreshock probability calculations in case of plural earthquakes in a cluster

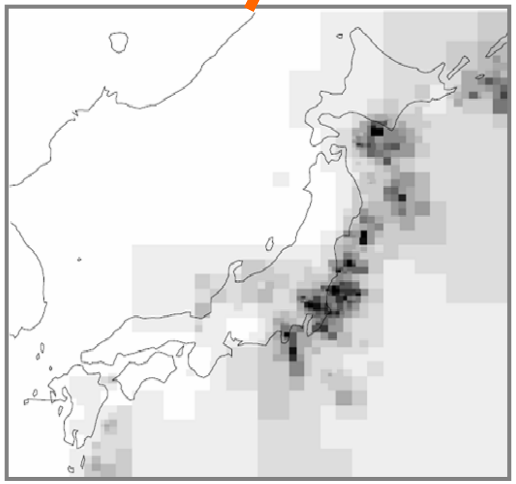
For plural earthquakes in a cluster, time differences t_{ij} (days), epicenter separation r_{ij} (km), magnitude difference g_{ij} are transformed into the unit cube

$$(t_{i,j}, r_{i,j}, g_{i,j}) \rightarrow (\tau_{i,j}, \rho_{i,j}, \gamma_{i,j}) \in [0,1]^3$$

Probability p_c is calculated sequentially by $p_c = e^{f_c} / \{ e^{f_c} + 1 \}$ where

$$f_c = \operatorname{logit}\{\mu(x_1, y_1)\} + \frac{1}{\#\{i<j\}} \sum_{i<j} \left[a_1 + \sum_{k=1}^{3} b_k \gamma_{i,j}{}^k + \sum_{k=1}^{3} c_k \rho_{i,j}{}^k + \sum_{k=1}^{3} d_k \tau_{i,j}{}^k \right]$$

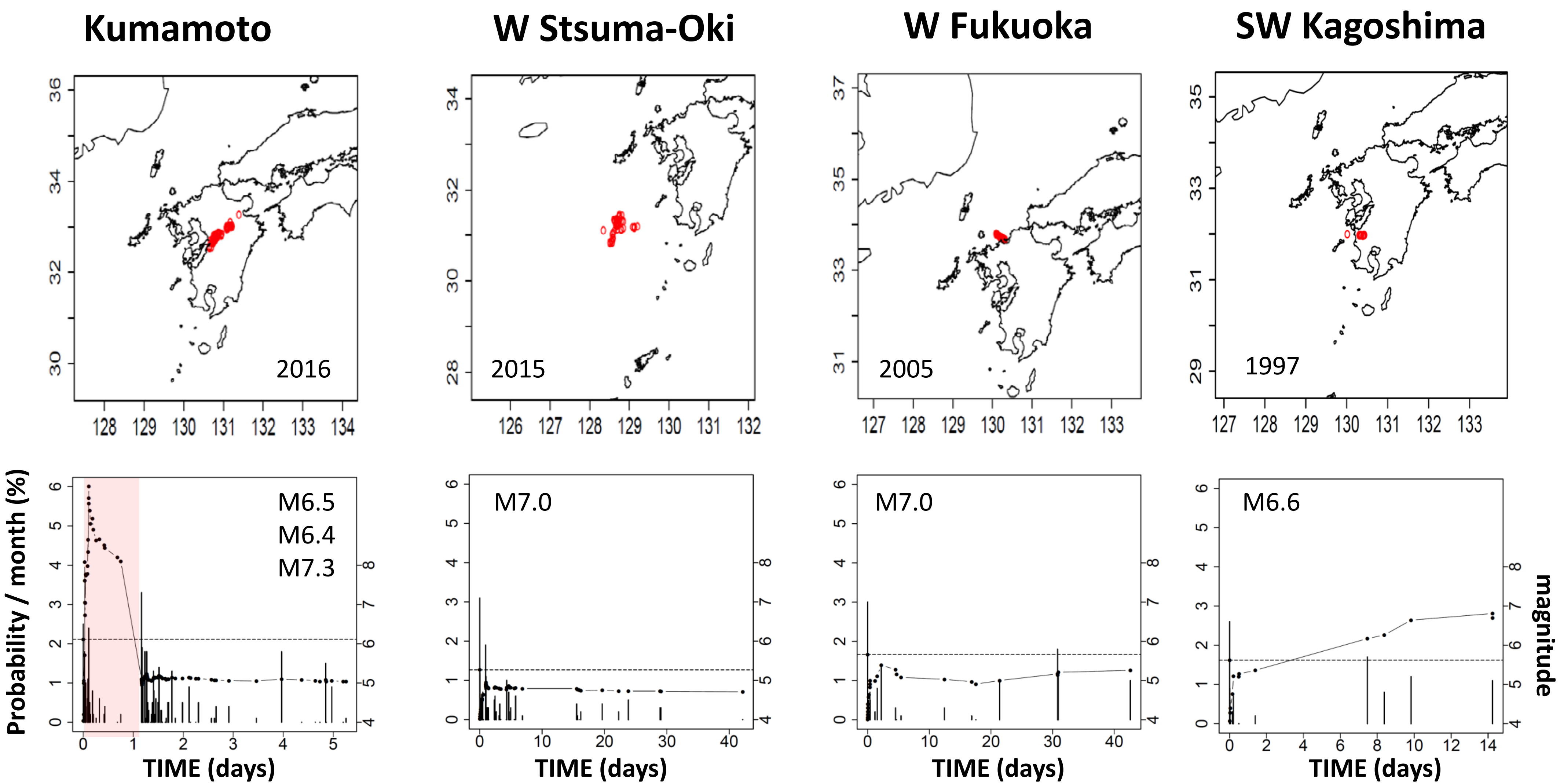
Here $\mu(x, y)$ indicates probability of initial earthquake at location (x,y), and the 2nd term calculates arithmetic mean of polynomials of the normalised space-time magnitude variables for all pairs of earthquakes $(i < j)$ in a cluster, where the coefficients a, b, c, d are as follows.



Ogata, Utsu and Katsura, 1996, *GJI*)

k	a_k	b_k	c_k	d_k
1	8.018	–33.25	–1.490	–10.92
2		62.77	2.805	295.09
3		–37.66	–2.190	–1161.5

Foreshock probability series



Short-term probabilities of $M \geqq 7$ earthquake in Kumamoto region

		30yrs	1yr	1mo.	1day
Secular S1	ETAS-μ & G-R law				0.00027 %
S2	#(M≧4) & G-R law				0.00058 %
Long-term A1	Futagawa Fault	0.9 %			0.000082 %
A2	Kumamoto Dist.	10.5 %			0.00096 %
A3	Central Kyushu	21.0 %			0.0019 %
Medium-term B1	Triggering		0.5 %		0.0014 %
B2	Quiescence		2.0 %		0.0055 %
Short-term C	**Foreshocks**			**5.0 %**	**0.17 %**

Short-term probabilities of $M \geqq 7$ earthquake in Kumamoto region

		30yrs	1yr	1mo.	per 1 day
Secular S1	ETAS-μ & G-R law				0.00027 %
S2	#(M≧4) & G-R law				0.00058 %
Long-term A1	Futagawa Fault	0.9 %			0.000082 %
A2	Kumamoto Dist.	10.5 %			0.00096 %
A3	Central Kyushu	21.0 %			0.0019 %
Medium-term B1	Triggering		0.5 %		0.0014 %
B2	Quiescence		2.0 %		0.0055 %
Short-term C	Foreshocks			5.0 %	0.17 %

Multiple elements probability formula (Utsu, 1977, *Zisin II*)

$$P(M \mid A \cap B \cap C,\ S) = \frac{1}{1 + \left(\frac{1}{P_A} - 1\right)\left(\frac{1}{P_B} - 1\right)\left(\frac{1}{P_C} - 1\right) \Big/ \left(\frac{1}{P_S} - 1\right)^{3-1}}$$

Legends

Secular:
S1 ETAS-mu & GR
S2 # $M \geqq 4$ & GR

Long-term:
A1 Futagawa Fault
A2 C. Kyushu/2
A3 C. Kyushu

Intermediate-term:
B1 Triggering
B2 Quiescence

Short-term:
C Foreshocks

Pr. per unit time	1day	3days	1week	1month
$P(M \mid A_1 \cap B_1 \cap C,\ S_1)$	0. 2 %	0.7 %	1.7 %	7.3 %
$P(M \mid A_1 \cap B_2 \cap C,\ S_1)$	1.0 %	2.9 %	6.6 %	23.9 %
$P(M \mid A_2 \cap B_1 \cap C,\ S_1)$	2.8 %	8.0 %	17.0 %	47.8 %
$P(M \mid A_2 \cap B_2 \cap C,\ S_1)$	10.4 %	25.9 %	45.1 %	78.6 %
$P(M \mid A_3 \cap B_1 \cap C,\ S_1)$	5.5 %	14.9 %	29.1 %	64.7 %
$P(M \mid A_3 \cap B_2 \cap C,\ S_1)$	18.8 %	41.1 %	62.2 %	88.0 %
$P(M \mid A_1 \cap B_1 \cap C,\ S_2)$	0.0 %	0.1 %	0.3 %	1.3 %
$P(M \mid A_1 \cap B_2 \cap C,\ S_2)$	0.2 %	0.5 %	1.2 %	5.1 %
$P(M \mid A_2 \cap B_1 \cap C,\ S_2)$	0.5 %	1.5 %	3.4 %	13.6 %
$P(M \mid A_2 \cap B_2 \cap C,\ S_2)$	2.0 %	5.7 %	12.4 %	38.8 %
$P(M \mid A_3 \cap B_1 \cap C,\ S_2)$	1.0 %	2.9 %	6.6 %	24.0 %
$P(M \mid A_3 \cap B_2 \cap C,\ S_2)$	3.9 %	10.8 %	22.1 %	55.9 %

Summary for Kumamoto case

Estimates of the probabilities of as a target prediction earthquake M7.0 or more

(per 1 $\deg^2$)~ Rectangle containing the aftershock area

Consecutive S: **Unconditional probability** ***Ps*** of the large earthquake per **90 years**

S1. Number of Mc4 & G-R law *b* = 1.0 (**0.00058% per day**)

S2. ETAS model μ & G-R law *b* = 1.0 (**0.0027% per day**)

Long-term A: A1. Futagawa Faults **0.9% per 30 years** (**0.000082% per day**)

A2. Central Kyushu region **21% per 30 years** (**0.0019% per day**)

A3. Kumamoto region **10.5% per 30 years** (**0.00095% per day**)

Medium-term B: B1. Triggering probability **0.5% per 1 year** (**0.0014% per day**)

B2. Relative quiescence probability **2% per 1 year** (**0.0055% per day**)

Short-term C: Foreshock Probab. of Apr. 14 M6.5 **=: 5%per month** (**0.17% per day**)

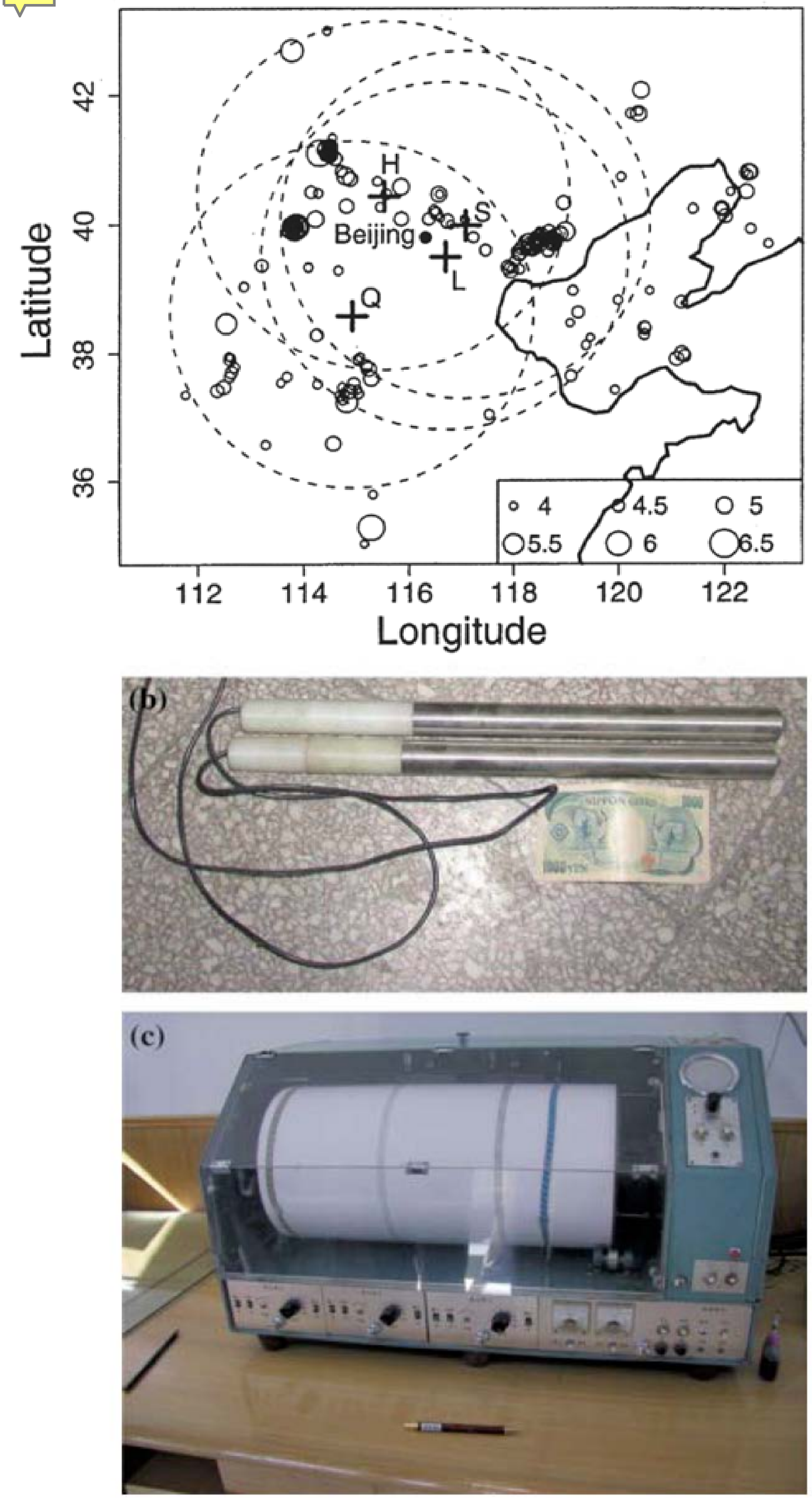
Multiple elements prediction formula

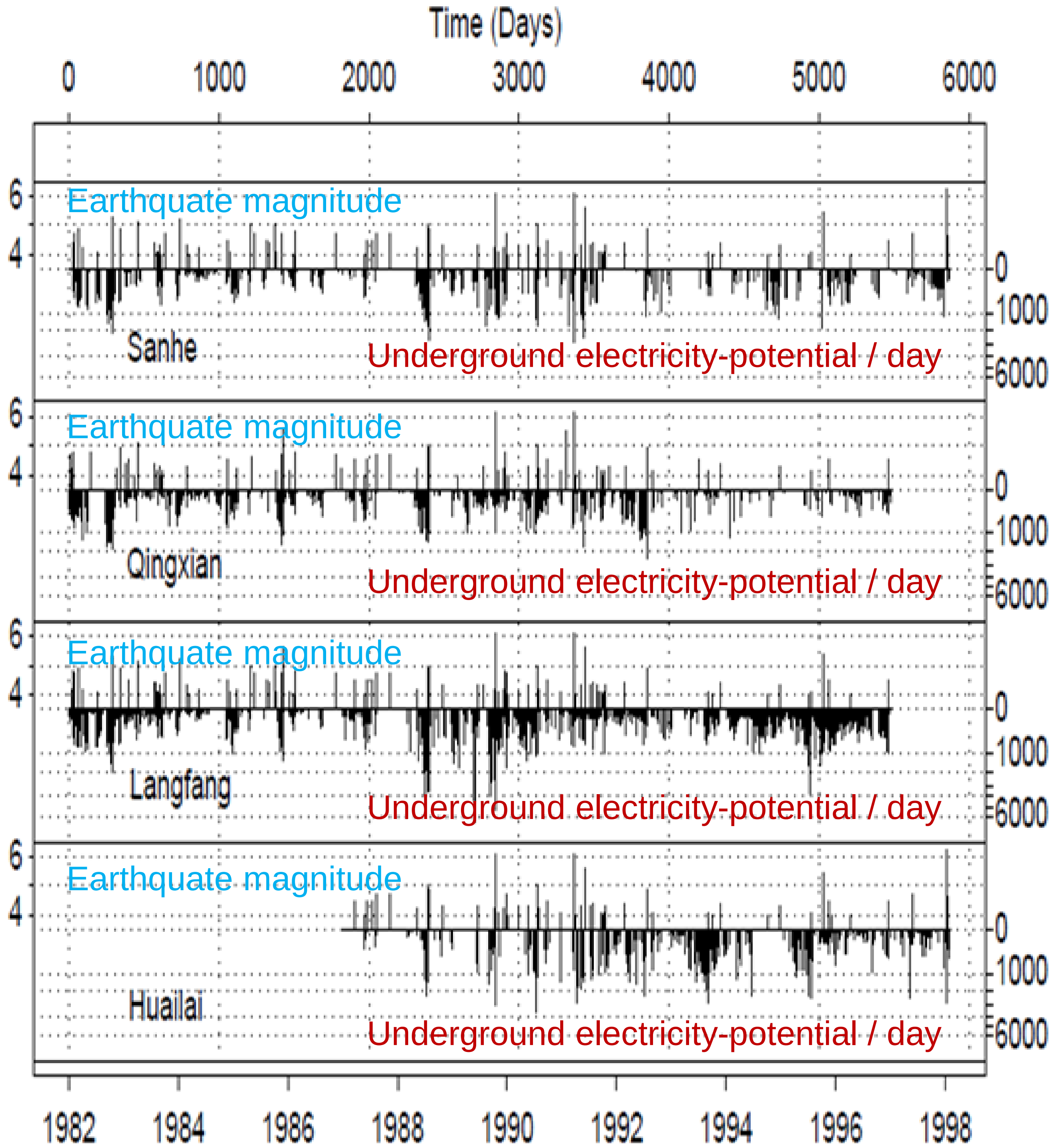
$$P = \frac{1}{1+\left(\frac{1}{P_A}-1\right)\left(\frac{1}{P_B}-1\right)\left(\frac{1}{P_C}-1\right)\Big/\left(\frac{1}{P_S}-1\right)^{3-1}}$$

	per 1 day	per 3 days	per 1 week	per 1 month
Probability	**0.007% ~ 20%,**	**0.016% ~ 41%,**	**0.03% ~ 62%,**	**0.2% ~ 88%**

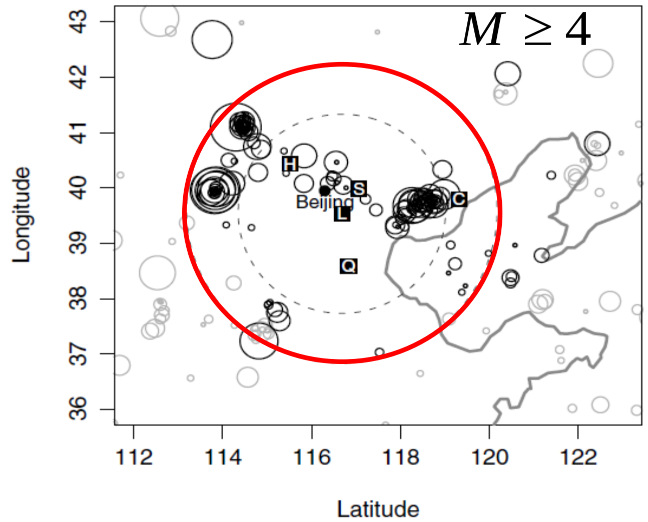
Some more comments about anomalies from other observations

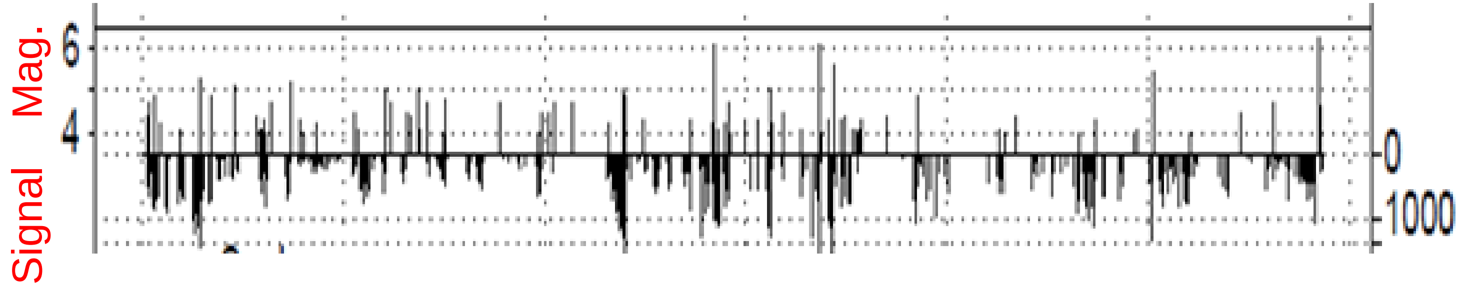
Zhuang et al. (2005, *PAGEOPH*)





Zhuang et al. (2005, *PAGEOPH*)

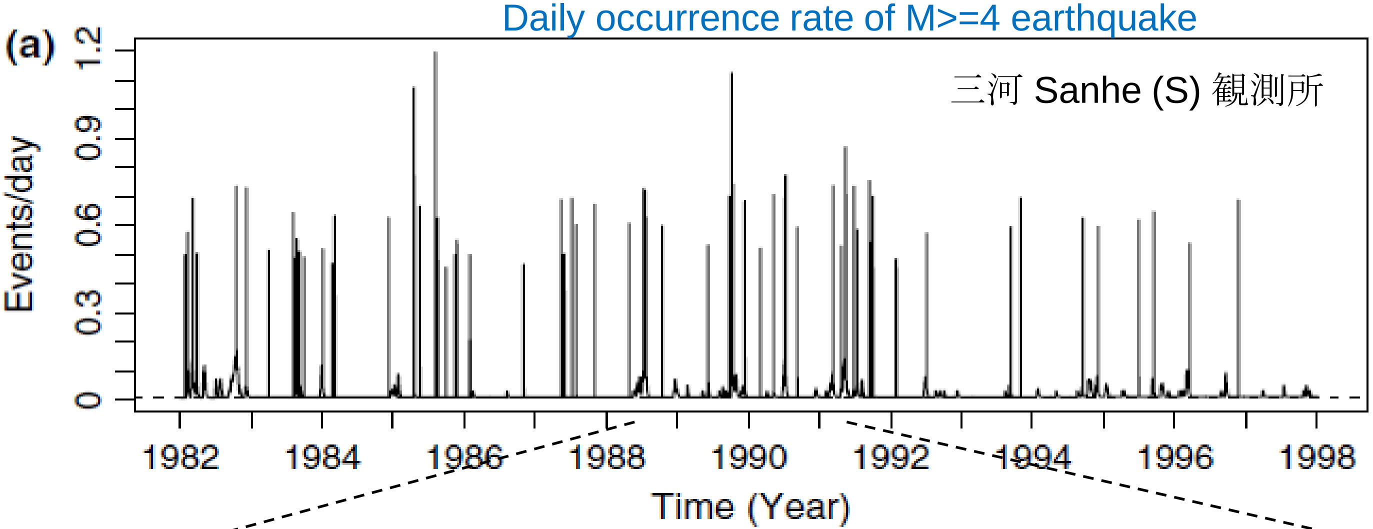


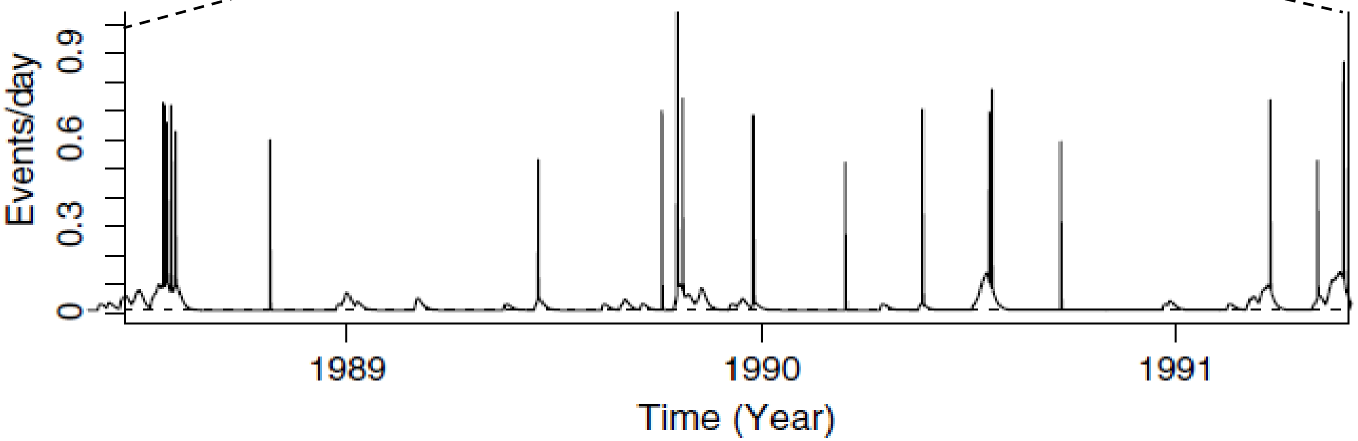


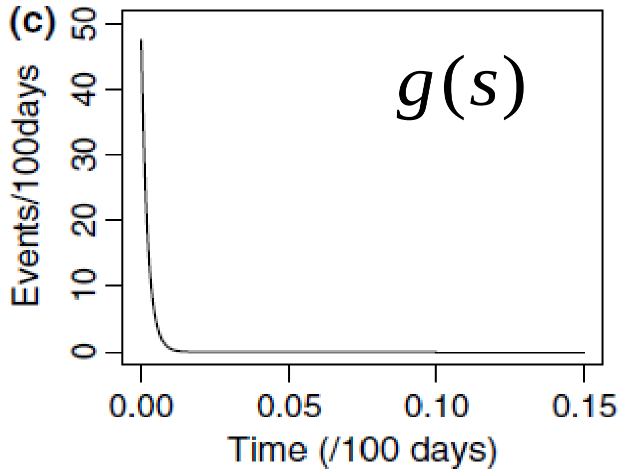
$$\lambda(t \mid H_t) = \mu + \int_0^t g(t-s)\,dN_s + \int_0^t h(t-s)\,dM_s$$

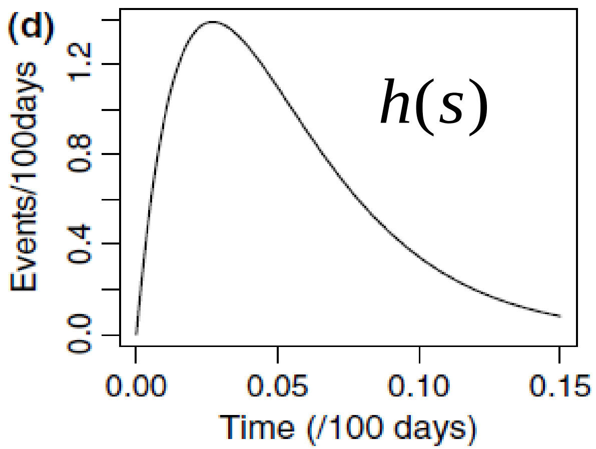
$$= \mu + \sum_{\{i;\, t_i<t\}} g(t-t_i) + \sum_{\{j;\, \tau_j<t\}} h(t-\tau_j)\ f(m_j)\Delta$$

Daily occurrence rate of M>=4 earthquake

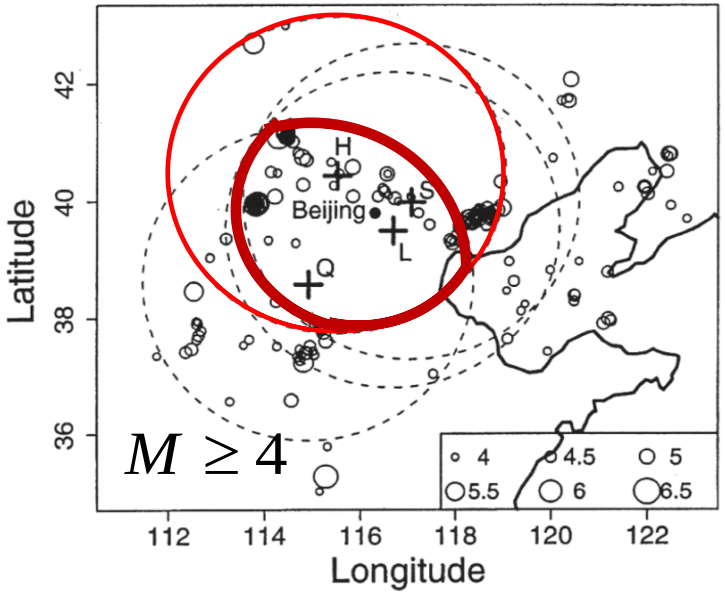


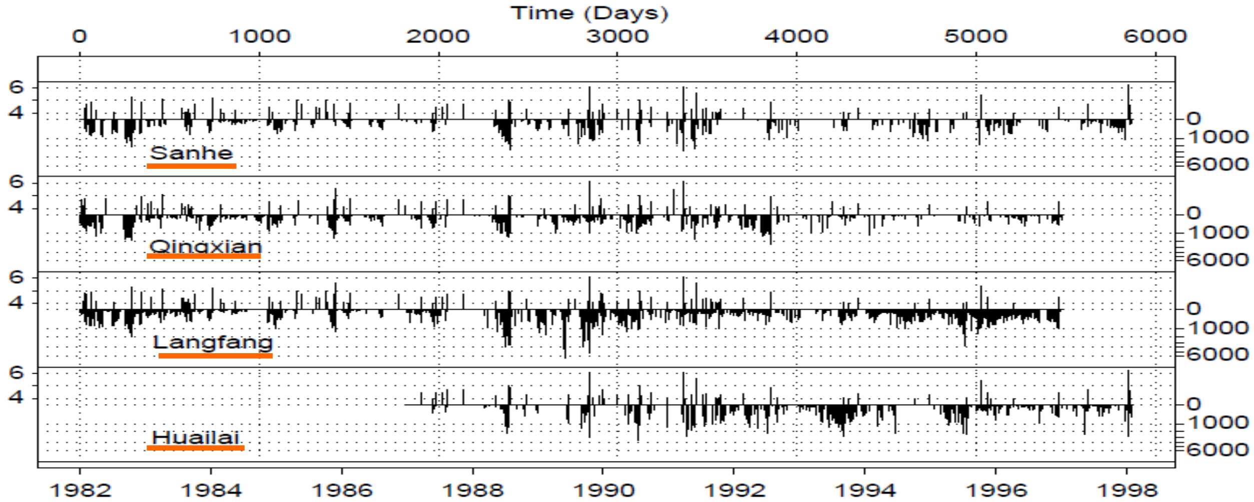




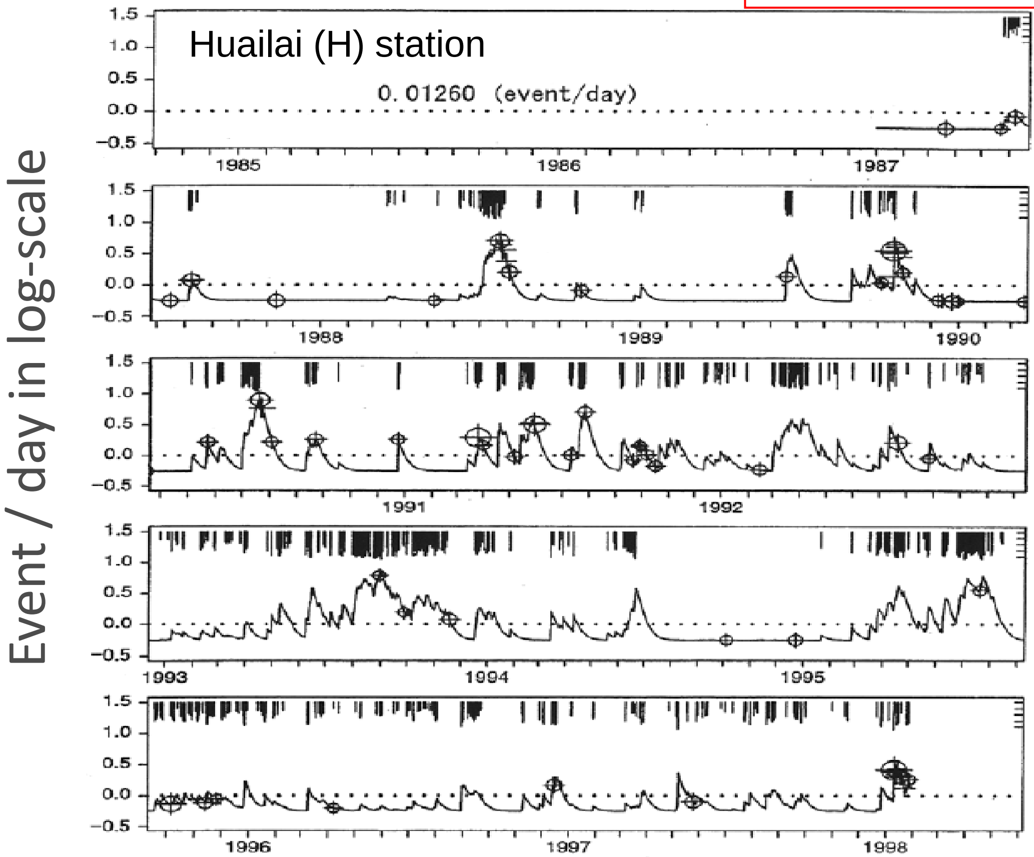


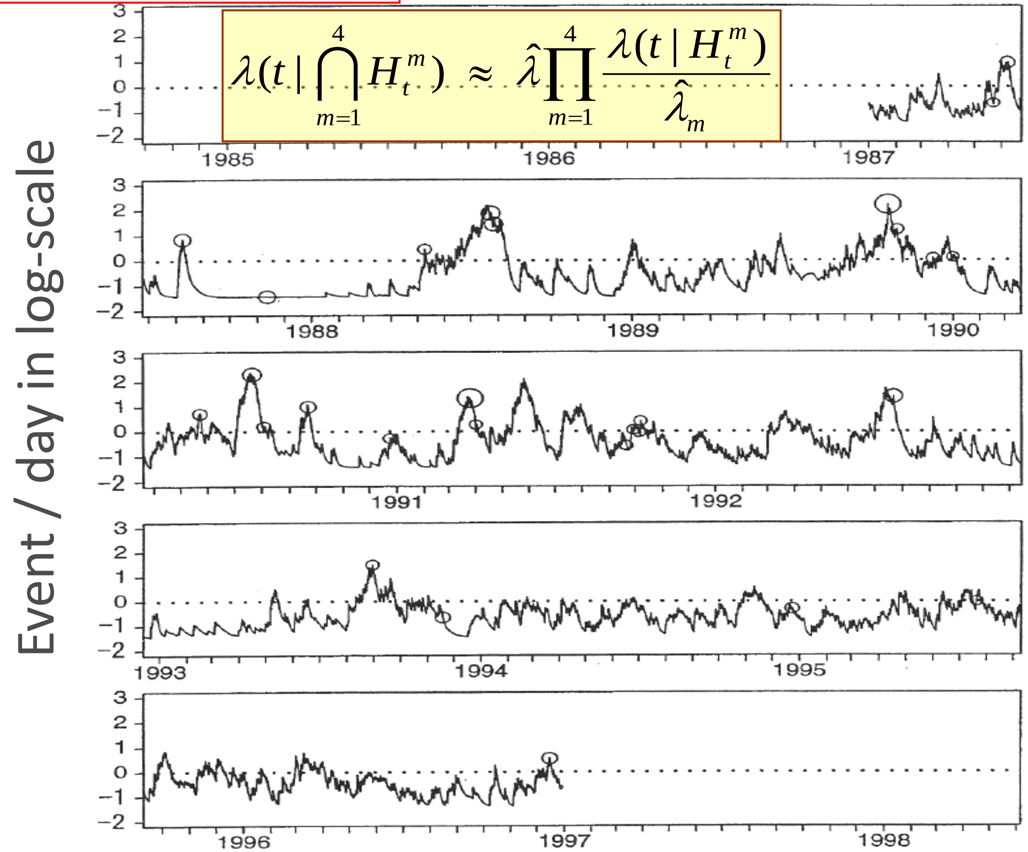
Zhuang & Ogata (2001, *RCCEP*)





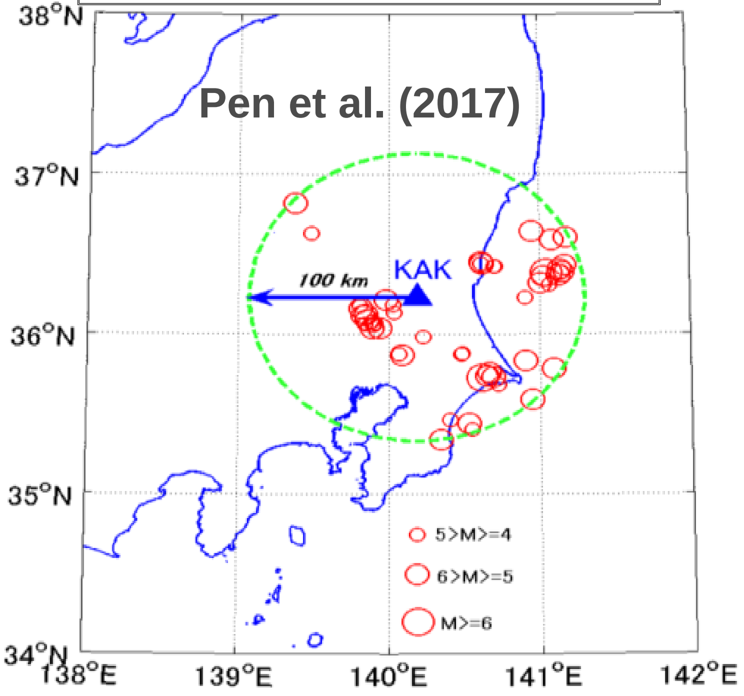
$$\lambda(t \mid H_t) = \mu + \int_S^t h(t-s)\xi(s)^a \, ds$$
$$= 0.00702 + \sum_{j=S}^{t} 0.000117\, e^{-0.142(t-j)}\, \xi_j^{0.69}$$

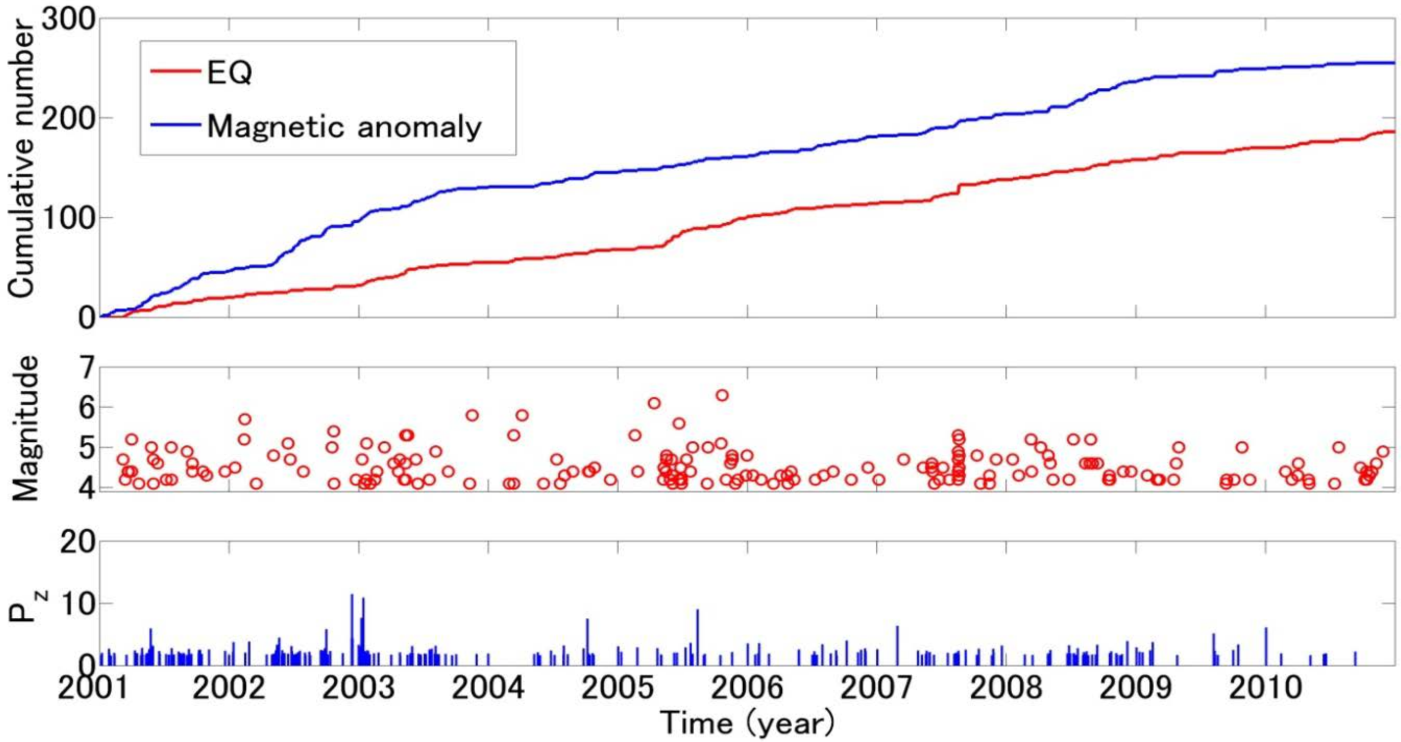




$$\lambda\left(t \mid \bigcap_{m=1}^{4} H_t^m\right) \approx \hat{\lambda} \prod_{m=1}^{4} \frac{\lambda(t \mid H_t^m)}{\hat{\lambda}_m}$$

Magnetic anomalies



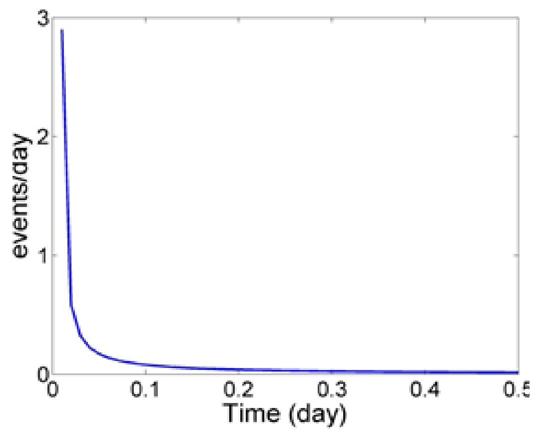


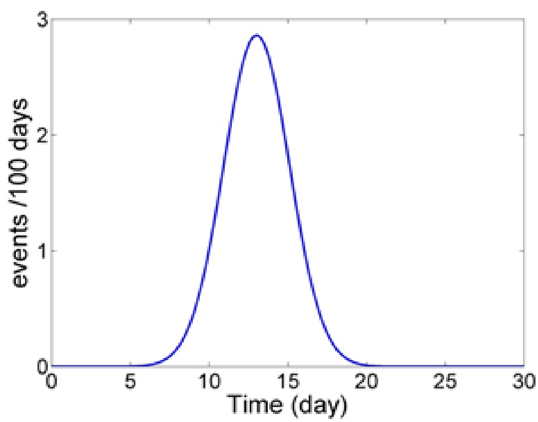
$$\lambda(t|H_t, F_t) = \mu + \lambda_S(t|H_t) + \lambda_E(t|F_t)$$

$$\lambda_S(t \mid H_t) = \sum_{t_i<t} g(t-t_i) = \sum_{t_i<t} \left(A e^{\alpha(M_i - M_c)} \frac{p-1}{c} \left(1 + \frac{t-t_i}{c}\right)^{-p} \right)$$

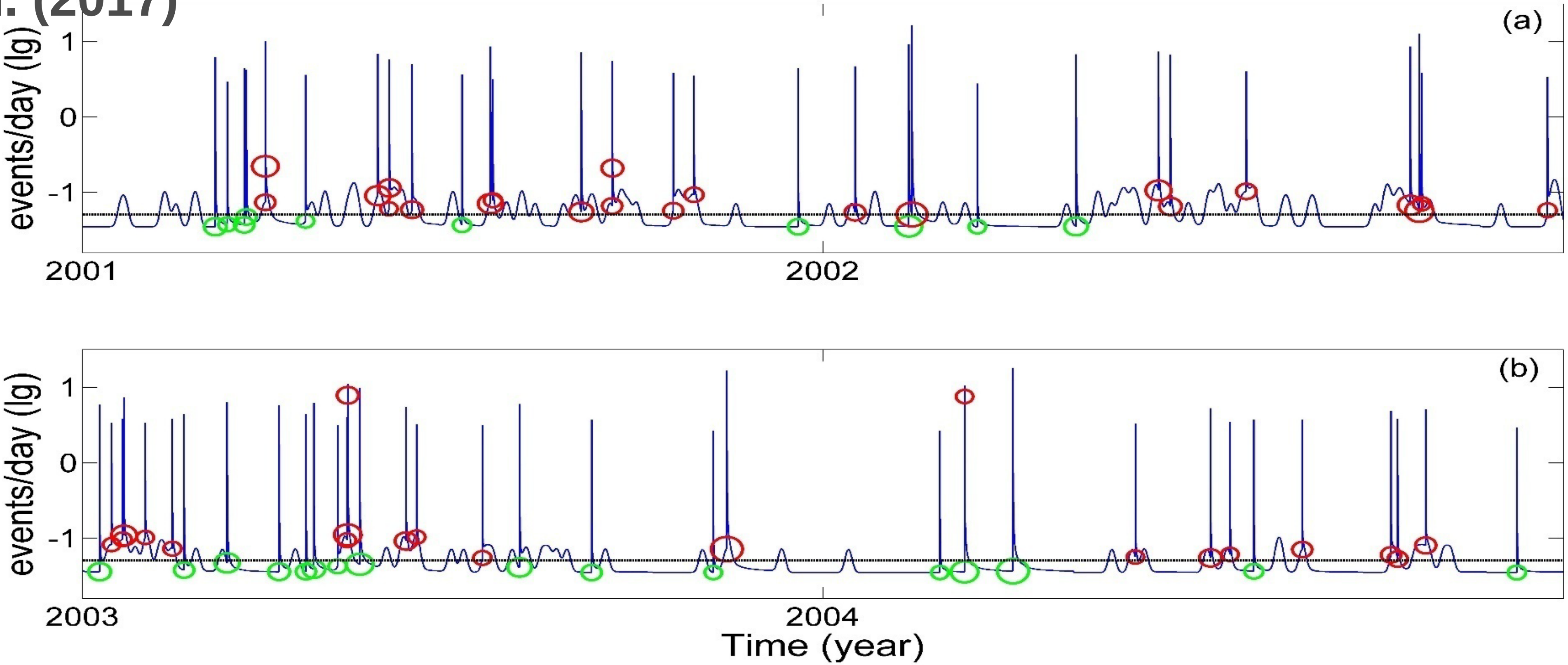
$$\lambda_E(t \mid F_t) = \sum_{\nu_i<t} f(t-\nu_i) = \sum_{\nu_i<t} B e^{\frac{-(t-\nu_i-L)^2}{2D^2}}$$

Model	#p	AIC	△AIC
Poisson model	1	1482.	0
ETAS model	5	1428.	−54
External model	4	1464.	−18
Combined model	8	1417.	−65





Pen et al. (2017)



Training: 2001 | 2002 | 2003 | 2004 | 2005

Forecasting: 2006 | 2007 | 2008 | 2009 | 2010

Vere-Jones, 1998

$$IG / N = (1/N)\ln\left(L/L_0\right)$$

EQ class	EQ number	External exciting	ETAS	Combined
		IG/N (vs. Poisson)	IG/N(vs. Poisson)	IG(vs. Poisson)
M >5.0	9	0.21	0.01	0.21
M>4.5	27	0.09	0.11	0.18
M>4.0	92	0.04	0.15	0.17

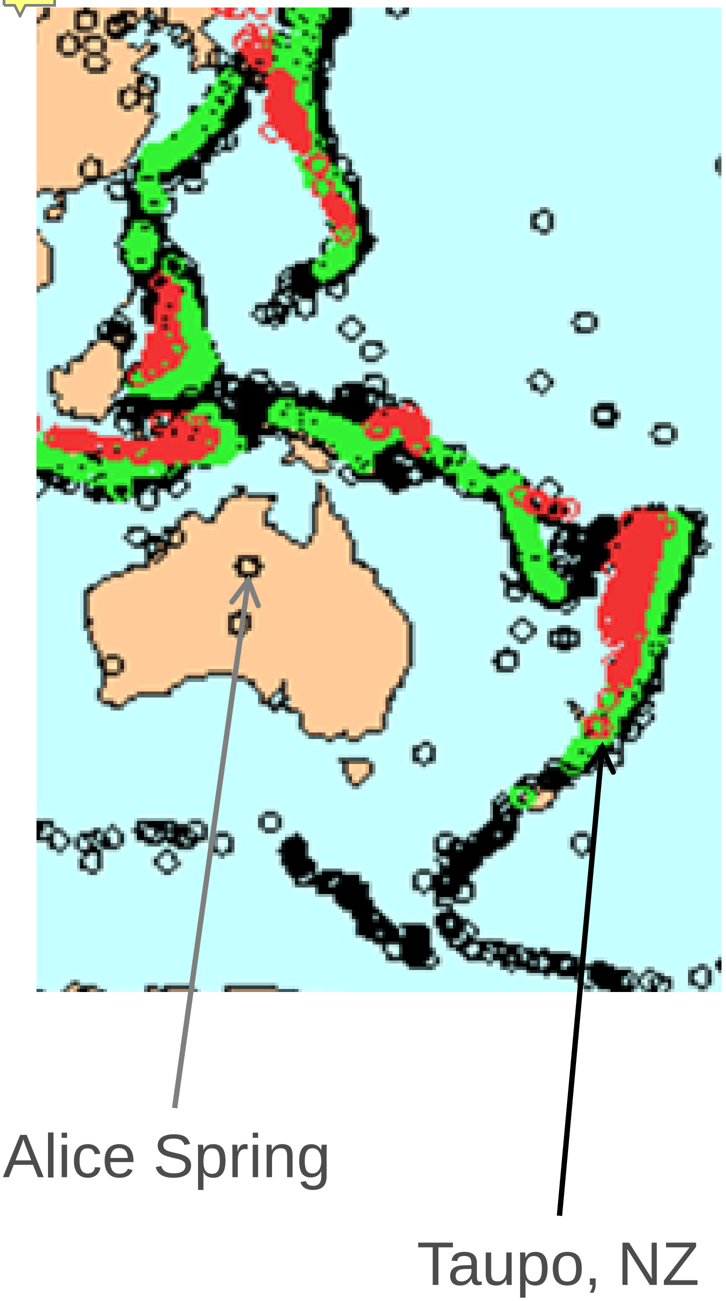
Aki's Approximation of Utsu formula

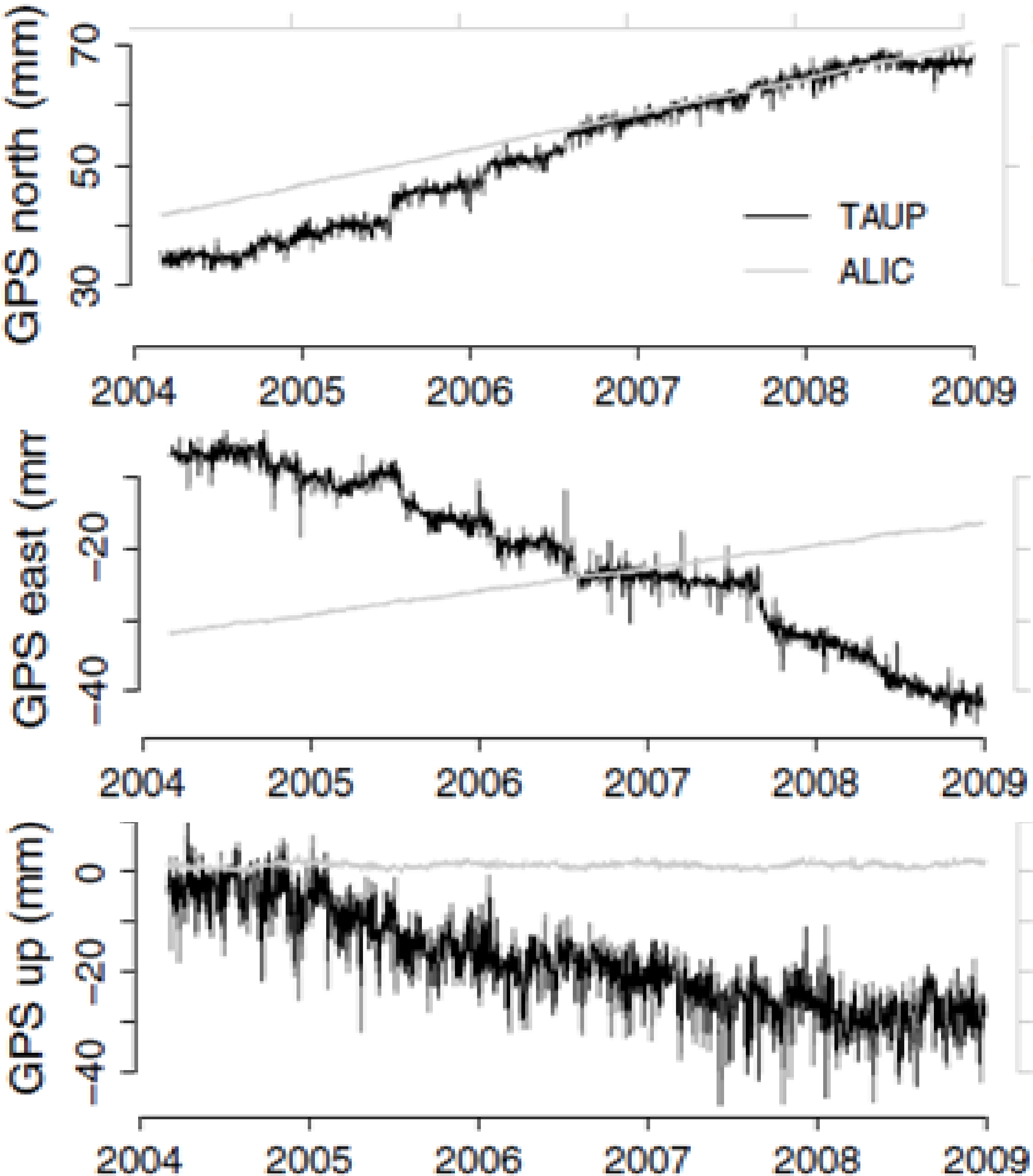
$$\frac{P(M \mid \bigcap_{n=1}^{N} A_n)}{P(M)} \approx \prod_{n=1}^{N} \frac{P(M \mid A_n)}{P(M)}.$$

Extension to space-time magnitude

$$\frac{\lambda(t,x,y,M \mid H_t, F_t^1, \cdots, F_t^K)}{\lambda_0(t,x,y,M \mid H_t)} = \prod_{k=1}^{K} \frac{\lambda_k(t,x,y,M \mid H_t, F_t^k)}{\lambda_0(t,x,y,M \mid H_t)}$$

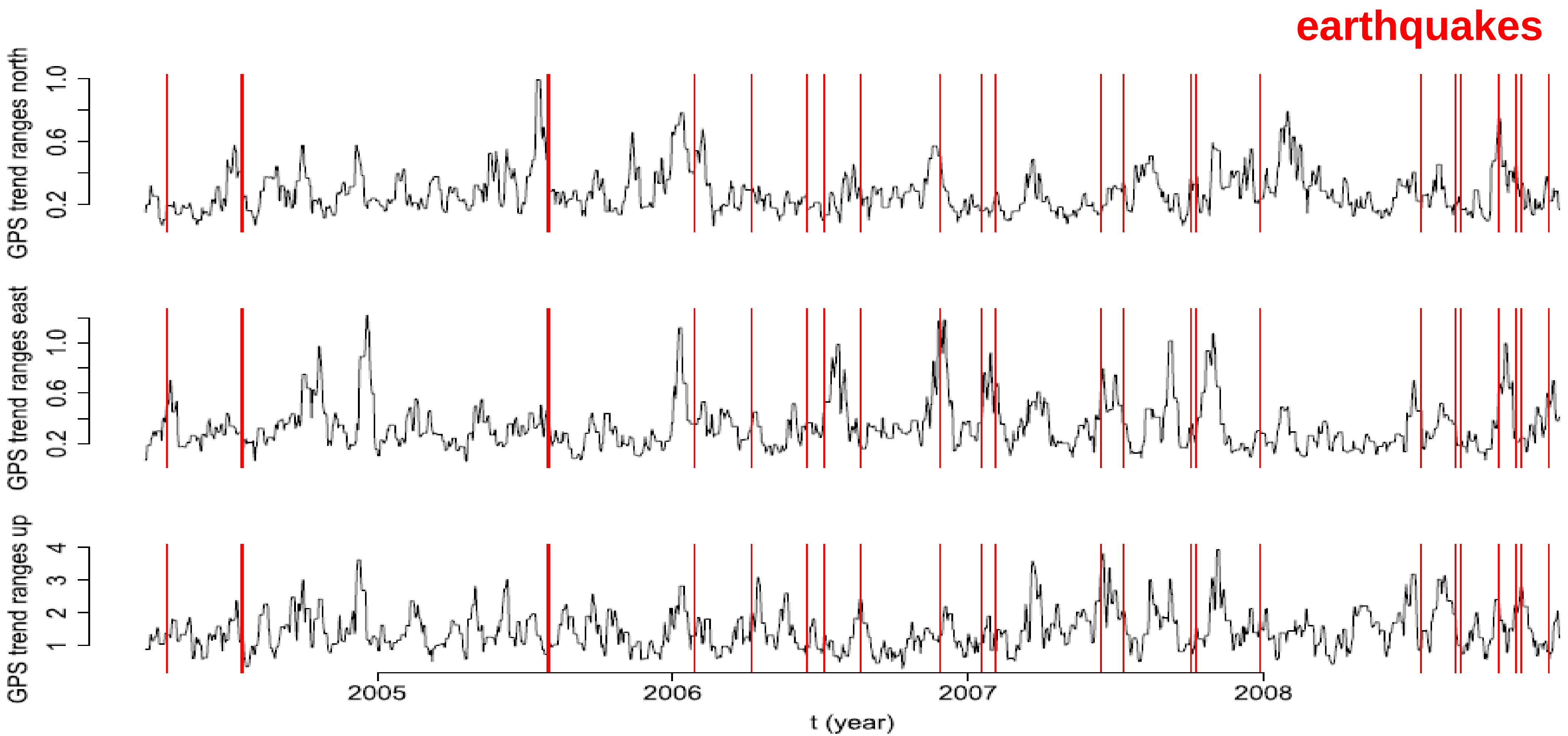
Desire to explore geodetic anomalies





Wang & Bebbington (2013)

fluctuation of base line expansion and contraction within the 10-days window



2004～2008, M≧5.1

Wang, Zhuang, Kato & Bebbington (2013, *GRL* Supplement)

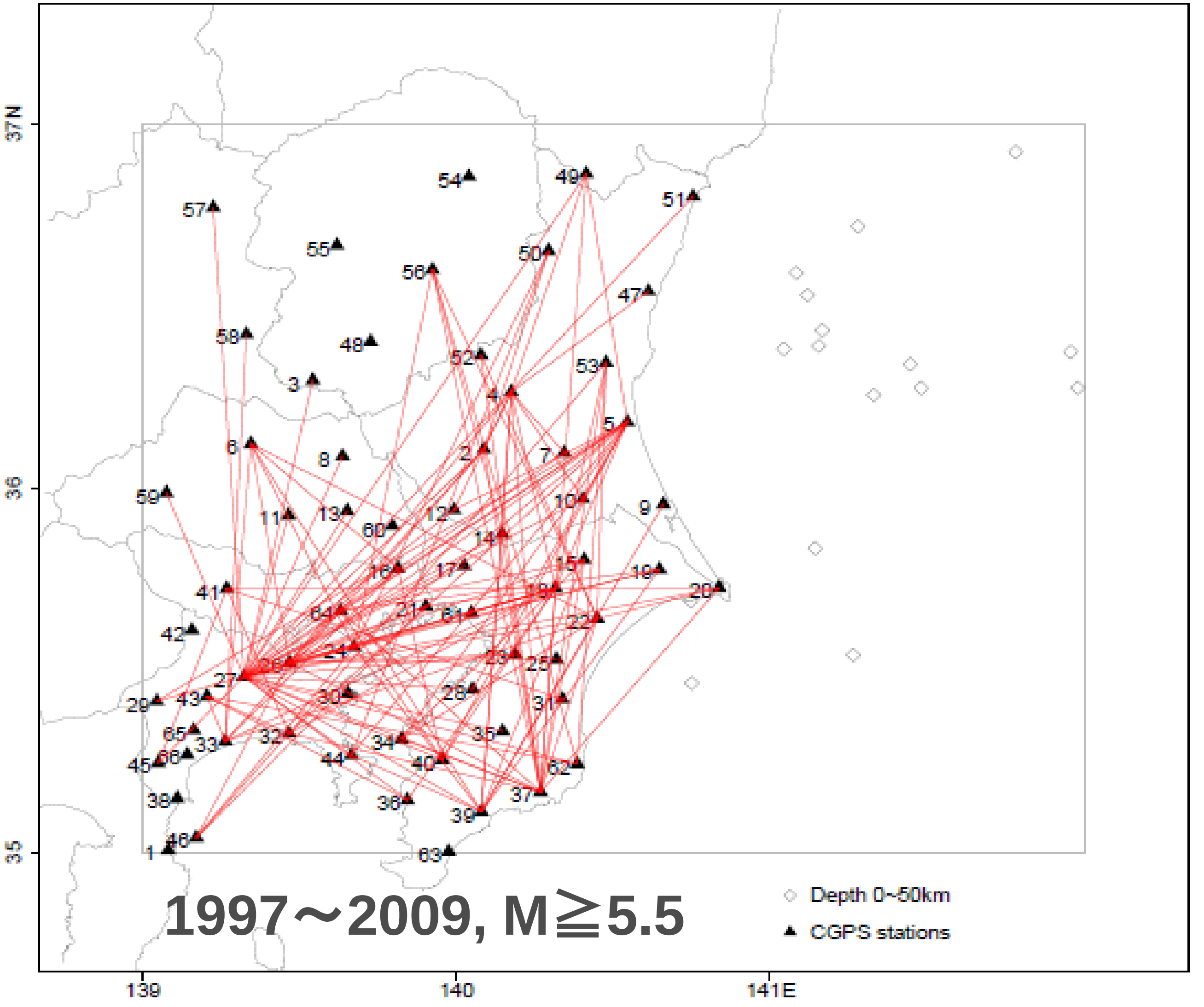


Figure 5: Map of the Kanto region with locations of the 66 GPS stations and earthquakes. Red lines indicate baselines which have probability gain over 2 with false alarm rate lower than 50%.

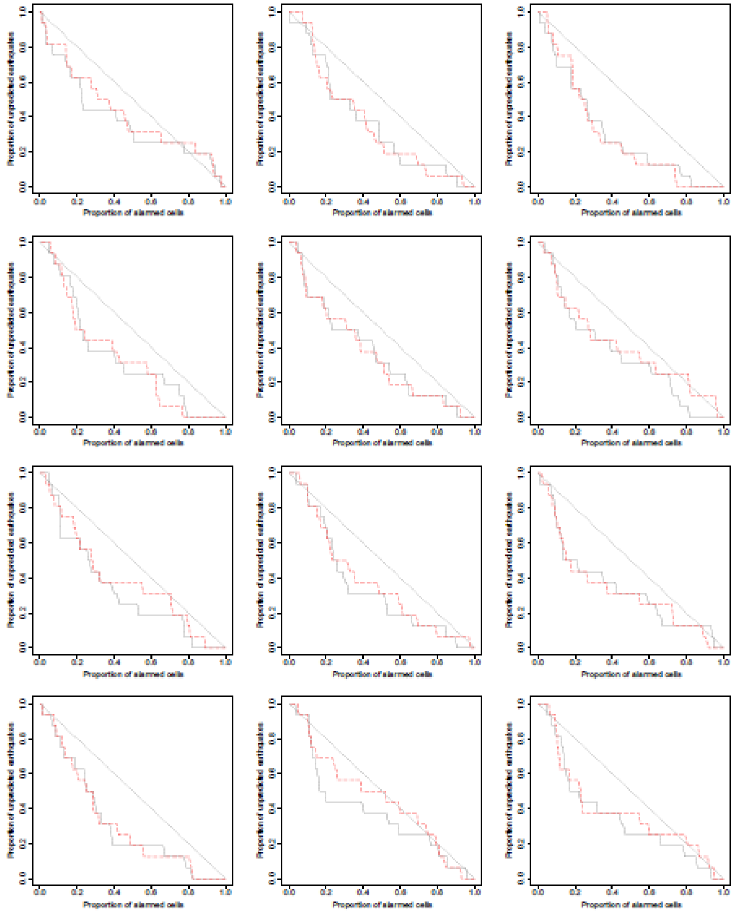
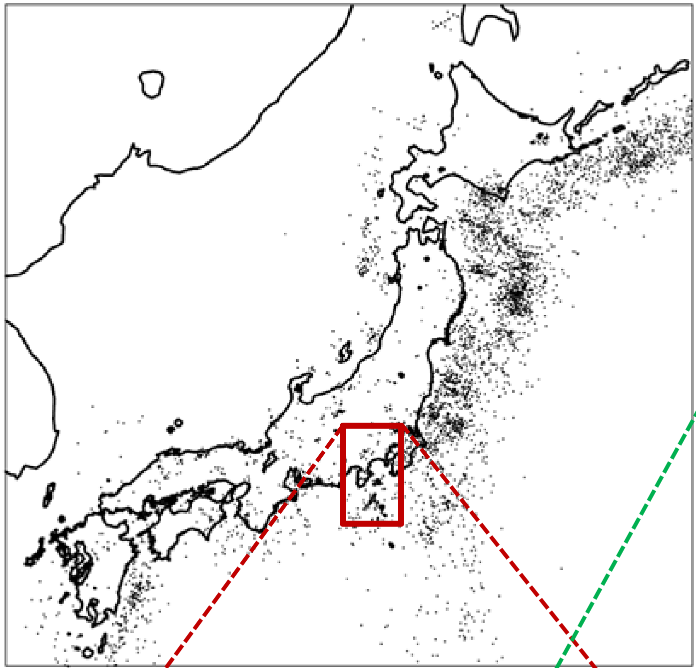


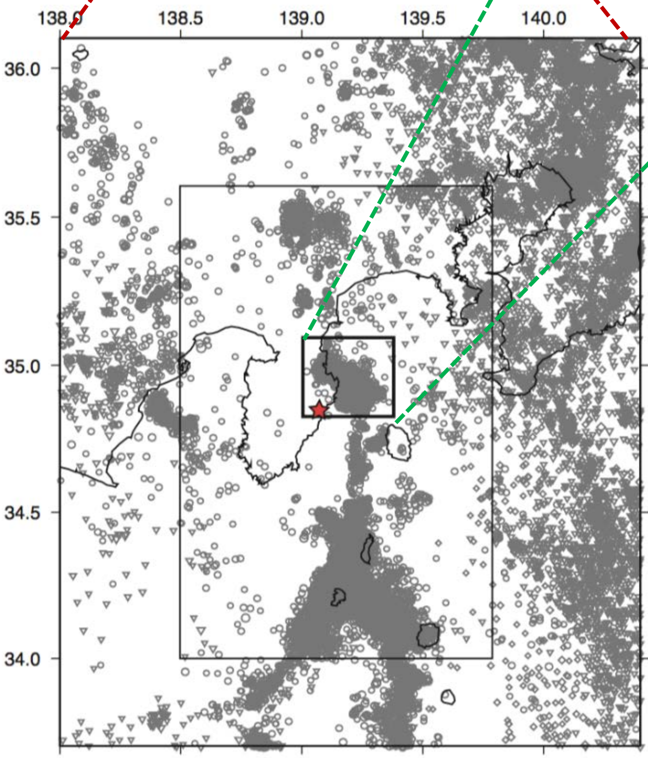
Figure 1: Molchan diagrams for baselines which show strong predictive power and have probability gain over 2 with false alarm rate lower than 50%. Black solid lines are for the alarm function $A^{(1)}$, and red dashed lines are for the alarm function $A^{(2)}$.

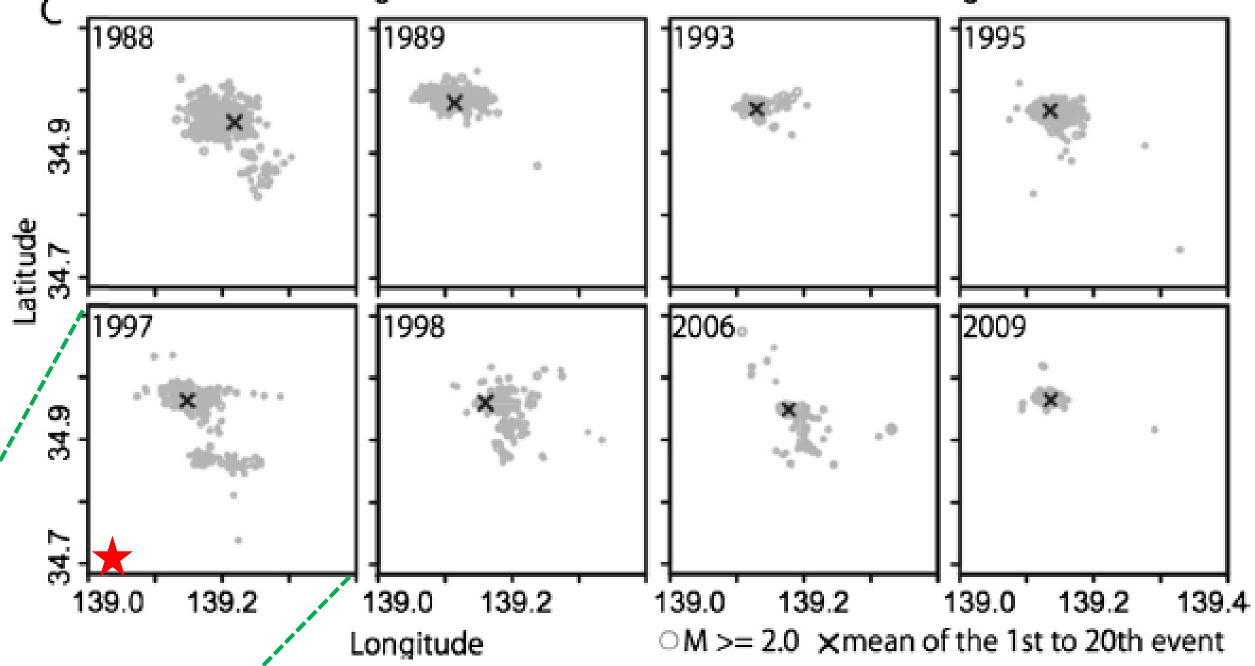
Horizontal only

Horizontal + vertical

Kumazawa et al. (2013, *EPSL*)

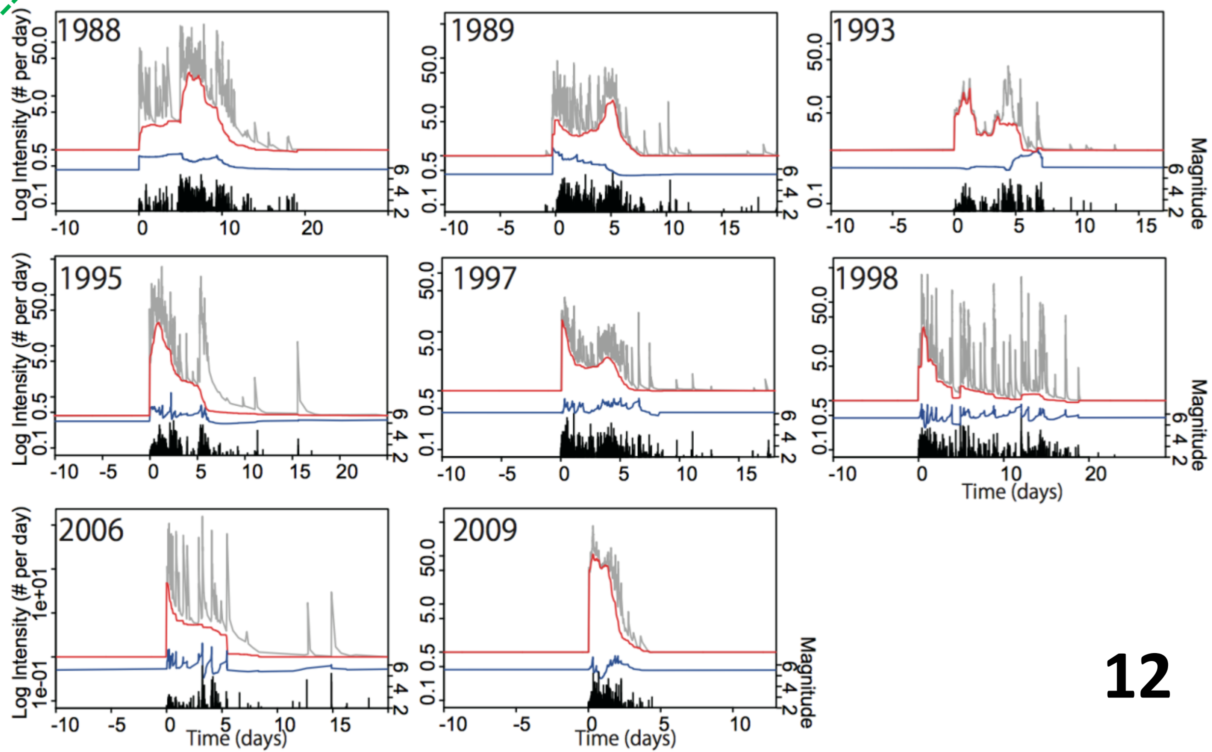






Non-stationary ETAS model :

$$\lambda(t \mid H_t) = \mu(t) + \sum_{\{i:\, t_i < t\}} \frac{K_0(t_i)\, e^{\alpha(M_i - M_c)}}{(t - t_i + c)^p}$$



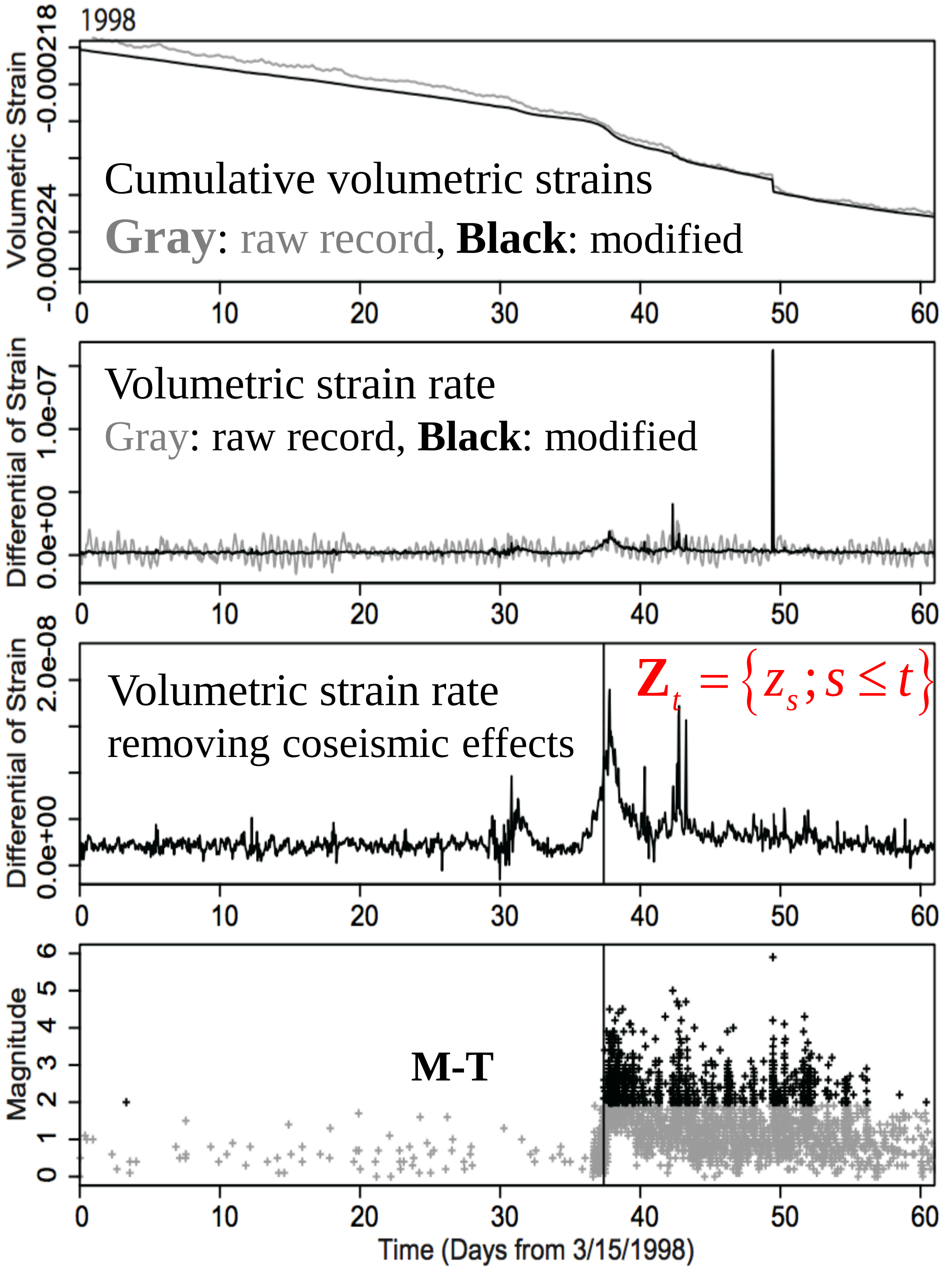
ETAS with causality data

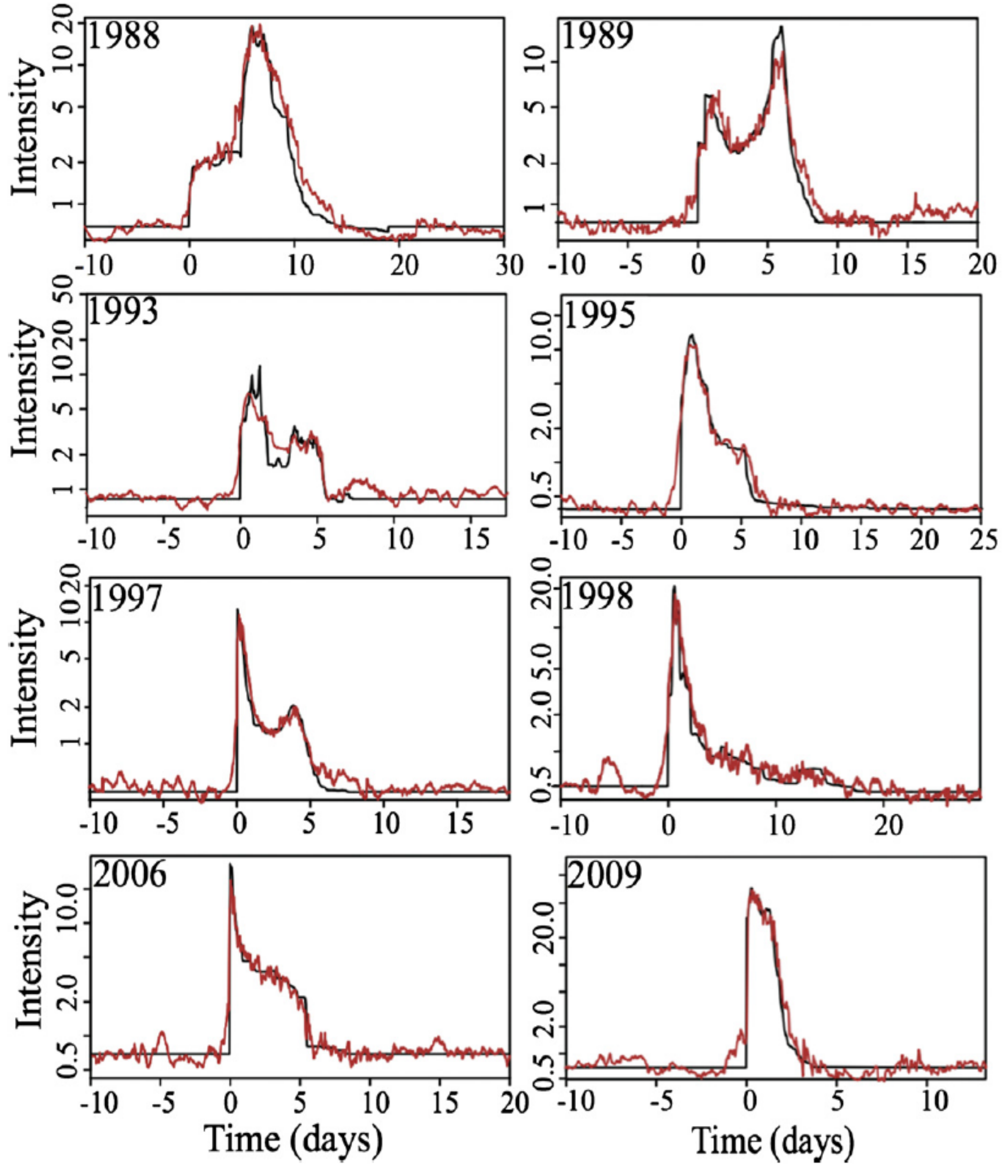
$$\lambda(t \mid H_t, x, y, Z_t) = \tilde{\mu}(t \mid x, y, Z_t) + \sum_{\{i: t_i < t\}} \frac{K_0(t_i) e^{\alpha(M_i - M_c)}}{(t - t_i + c)^p}$$

where

$$\tilde{\mu}(t \mid x, y, \mathbf{Z}_t) = \left(a_1 + \frac{a_2}{a_3 + d(x, y)} \right) \int_{-\infty}^{t} e^{-\sigma(t-s)} z_s \, ds$$

Kumazawa et al (2016, EPSL)





Issues and further research for prospective probability forecasting

（1）There have been very many cases where no precursory phenomena were observed preceding large earthquakes = very low prediction rate

（2）To improve the low prediction rate, we need to attain sensitive detection of abnormal phenomenon by the statistical residual analysis using the standard model for the observed data in every monitoring field.

（3）By many such anomalies even if each are low hitting rate, we can apply the Utsu formula or its extended version for a higher hitting rate.

（4）Many efforts to obtain the probability gains are required after a lot of subtle abnormal values become apparent by careful statistical diagnostic analysis.

Independent

$$P\left(\bigcap_{n=1}^{N} A_n \mid M\right) = \prod_{n=1}^{N} P(A_n \mid M) \quad \Rightarrow \quad P\left(M \mid \bigcap_{n=1}^{N} A_n\right) = \left[1 + \prod_{n=1}^{N}\left\{P(M \mid A_n)^{-1} - 1\right\} \Big/ \left\{P(M)^{-1} - 1\right\}^{N-1}\right]^{-1}$$

$$p_n = \frac{1}{(1+e^{-f_n})} \quad \Leftrightarrow \quad f_n = \operatorname{logit}(p_n) = \log \frac{p_n}{1-p_n} \qquad (n = 1, 2, \cdots, N)$$

$$\operatorname{logit} P(M \mid \bigcap_{n=1}^{N} A_n) = \sum_{n=1}^{N} \operatorname{logit} P(M \mid A_n) - (N-1)\operatorname{logit} P(M),$$

$$\operatorname{logit} P(M \mid \bigcap_{n=1}^{N} A_n) - \operatorname{logit} P(M) = \sum_{n=1}^{N} \left\{\operatorname{logit} P(M \mid A_n) - \operatorname{logit} P(M)\right\}$$

$$= \sum_{n=1}^{N} \left\{ f_n - f_0 \right\},$$

$$\operatorname{logit} P(M \mid \bigcap_{n=1}^{N} A_n) - \operatorname{logit}(M) = \varphi(f_1 - f_0,\; f_2 - f_0,\; \ldots,\; f_N - f_0)$$

Ogata, Utsu & Katsura, *GJI*, 1996

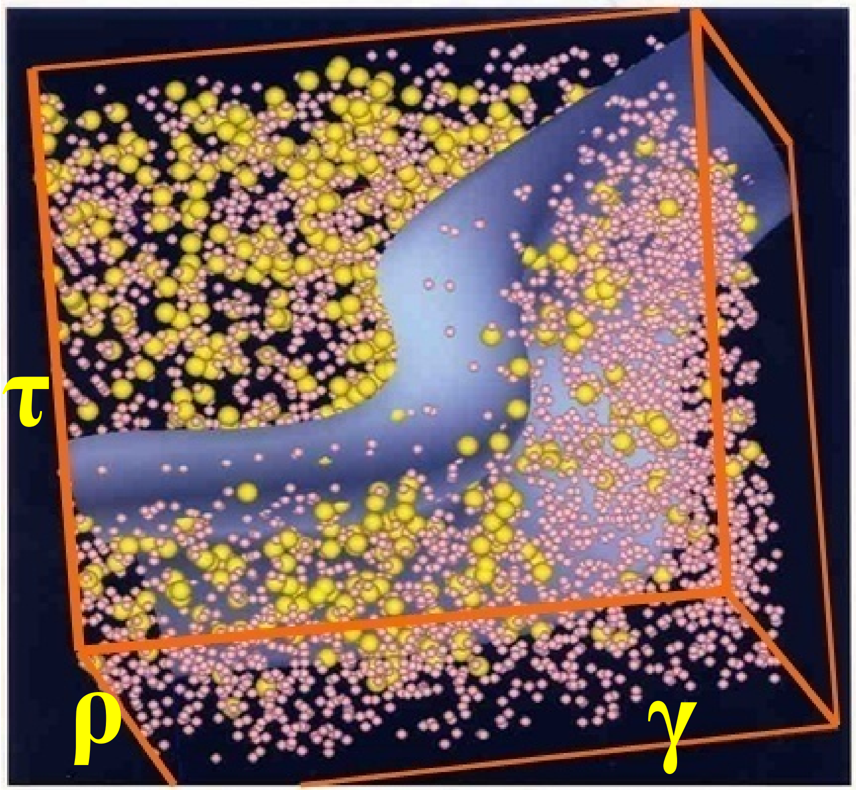
Time difference: $t^{c}_{i,j} = t^{c}_{j} - t^{c}_{i}$

Epicenter separation: $r^{c}_{i,j} = \sqrt{(x^{c}_{i} - x^{c}_{j})^2 + (y^{c}_{i} - y^{c}_{j})^2}$

Magnitude difference: $g^{c}_{i,j} = M^{c}_{j} - M^{c}_{i}$

$$(\tau^{c}_{i,j},\ \rho^{c}_{i,j},\ \gamma^{c}_{i,j}) \in [0,1]^3$$

unit cube



① **Time Interval Transformation:** $t \rightarrow \tau$

$$\tau = \begin{cases} 0 & \text{for} \quad t \le 0.01 \\ \log(100t)/\log(3000) & \text{for} \quad 0.01 < t \le 30 \\ 1 & \text{for} \quad 30 \le t \end{cases}$$

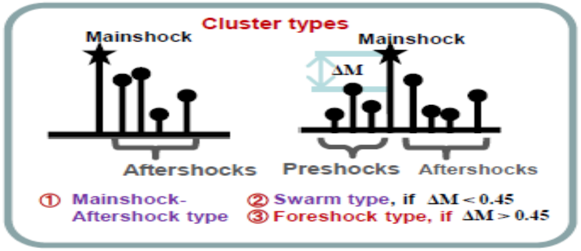
② **Epicenter Separation Transformation:** $r \rightarrow \rho$

$$\rho = 1 - \exp\{-\min(r, 50)/20\}$$

③ **Magnitude Difference Transformation:** $g \rightarrow \gamma$

$$\gamma = \begin{cases} (2/3)\exp\{g/\sigma_1\} & \text{for } g \le 0 \\ (2/3) + (1/3)[1 - \exp\{-g/\sigma_2\}] & \text{for } g > 0 \end{cases}$$

where $\sigma_1 = 6709,\ \sigma_2 = 0.4456$



$$f = \mathrm{logit}(p) \triangleq \log\left\{\frac{p}{1-p}\right\} \Leftrightarrow p = \frac{e^f}{1+e^f}$$

$$\mathrm{logit}(p_{\gamma,\rho,\tau}) \triangleq \log\left\{\frac{1-p_{\gamma,\rho,\tau}}{p_{\gamma,\rho,\tau}}\right\} = a + \sum_{k=1}^{3} b_k \gamma^k + \sum_{k=1}^{3} c_k \rho^k + \sum_{k=1}^{3} d_k \tau^k$$

Hint of statistical modeling of probability prediction; Ogata, Utsu & Katsura (1995, GJI)

$$p = \frac{1}{1+e^{f}} \quad \Leftrightarrow \quad f \equiv \textbf{logit}\ p \equiv \log \frac{1-p}{p}$$

$$\textbf{logit p}_\textbf{c} = \textbf{logit Pr}\left[\textbf{Foreshock} \ \middle| \left\{ \tau_{ij}^{c}, \rho_{ij}^{c}, \gamma_{ij}^{c} \right\} \right] = \mu_0 + E\left[f_\theta \left(\tau_{ij}^{c}, \rho_{ij}^{c}, \gamma_{ij}^{c} \right) \right]$$

$$\textbf{logit p}_\textbf{c} = \textbf{logit Pr}\left[\textbf{Foreshock} \ \middle| \left\{ \tau_{ij}^{c}, \rho_{ij}^{c}, \gamma_{ij}^{c} \right\} \right]$$

$$= \mu_0 + f_\theta \left(E\left[\tau_{ij}^{c} \right], E\left[\rho_{ij}^{c} \right], E\left[\gamma_{ij}^{c} \right] \right)$$

$$f_\theta(\tau, \rho, \gamma) = \sum_{l=0}^{L} \sum_{m=0}^{M} \sum_{n=0}^{N} a_{l,m,n} \tau^{l} \rho^{m} \gamma^{n},$$

where $\theta = (\{a_{l,m,n}\})$, and also its special case,

$$f_\theta(\tau, \rho, \gamma) = f_{\theta_1}(\tau) + f_{\theta_2}(\rho) + f_{\theta_3}(\gamma) = \sum_{l=0}^{L} b_l \tau^{l} + \sum_{m=0}^{M} c_m \rho^{m} + \sum_{n=0}^{N} d_n \gamma^{n}$$

Independent phenomena

Multiple Prediction Formula (Utsu, 1978)

F := { Ongoing events will be **FORESHOCKS** }

$$\mathbf{logit}\ p_c \equiv$$

logit *Prob*{*F* | temporal feature of a cluster}

+ logit *Prob*{*F* | spatial feature of a cluster }

+ logit *Prob*{*F* | magnitude sequential feature}

$$\text{logit}(p_{c|n}) = \frac{1}{\#\{i < j \le n\}} \sum_{i<j\le n} \left(\mu_0 + \sum_{k=1}^{3} b_k \gamma_{i.j}^k + \sum_{k=1}^{3} c_k \rho_{i.j}^k + \sum_{k=1}^{3} d_k \tau_{i.j}^k \right),$$

Table. 4. Coefficients of the polynomials of logistic models.

Data	$l=m=n$;	$\mu_0+a_{0,0,0}$	b_l	c_m	d_n
MBC 1926~1993	1	5.932	−22.20	−3.070	.0904
	2	—	42.80	9.664	−.2036
	3	—	−27.16	−7.814	.0973
SLC 1926~1993	1	8.018	−33.25	−1.490	−.3643
	2	—	62.77	2.805	.3278
	3	—	−37.66	−2.190	−.0430

Ogata, Utsu & Katsura (1996, *GJI*)

Multiple Prediction Formula (Utsu, 1978)

This is the probability of foreshocks by a logit model of the 4 independent features of location, epicenter separations, time and magnitude differences. Namely,

F := { Ongoing events will be FORESHOCKS }

$$\mathbf{logit}\{p_c(x,y)\} \equiv$$

= logit *Prob*{*F* | location of the first event }

+ logit *Prob*{*F* | temporal feature of a cluster}

+ logit *Prob*{*F* | spatial feature of a cluster }

+ logit *Prob*{*F* | magnitude sequential feature}

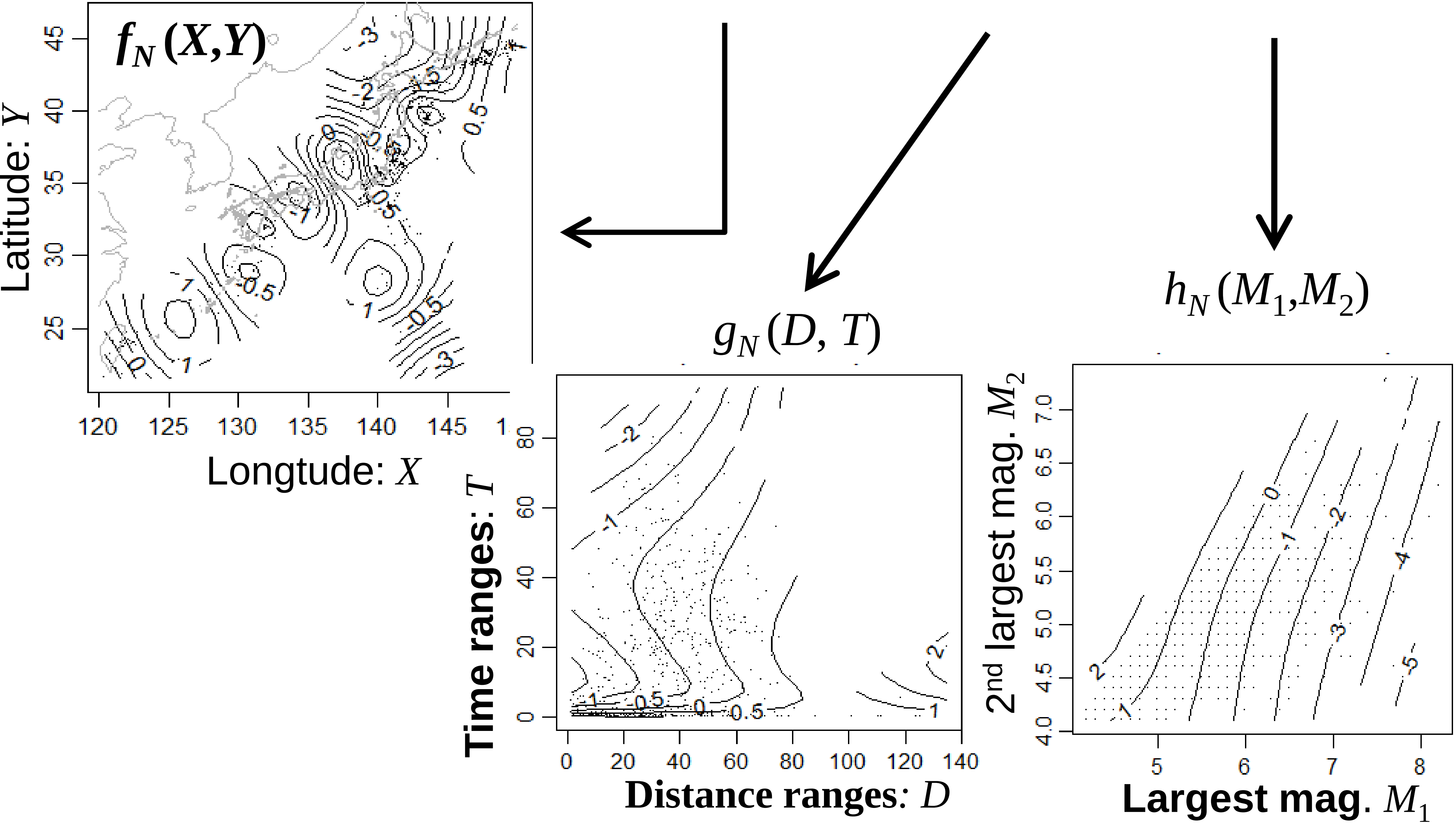
$$= \mathrm{logit}\{\mu(x_1, y_1)\} + \mathrm{E}_{\tau,\rho,\gamma}\left[\sum_{l=0}^{3} b_l \tau^l + \sum_{m=0}^{3} c_m \rho^m + \sum_{n=0}^{3} d_n \gamma^n\right]$$

Here $\mathrm{E}_{\tau,\rho,\gamma}[\cdot]$ indicates the expectation of the function values with respect to the Transformed time-differences, $\tau_{i,j}$, epicenter separations, $\rho_{i,j}$, and magnitude differences, $\gamma_{i,j}$ for all pairs *i, j* (*i* < *j*) of earthquakes within the cluster at the current time. The coefficients were estimated from the JMA data for the period from 1926 till 1994 (Ogata et al., 1996). The table below shows the estimated coefficients

P_N = **Probability of the largest earthquake of M > M_1**

$$\mathrm{Logit}\{P_N(\mathbf{M}|\,X,Y,D,T,M_1,M_2)\} = \boldsymbol{f_N}(\mathbf{M}|\,X,Y) + \boldsymbol{g_N}(\mathbf{M}|\,D,T) + \boldsymbol{h_N}(\mathbf{M}|\,M_1,M_2)$$

Long. & Lat. **Distances&times** **magnitudes**



Logistic function: logit(p) = log{p/(1－p)}

- 訓練データにロジスティック回帰（一般化加法モデル）を適用し、地震群が前震であるか（本震が続くか）を確率的に判別させる。

$$\mathrm{logit}(\underline{P_N(X,Y,D,T,M_1,M_2)}) = \underline{f_N(X,Y)} + \underline{g_N(D,T)} + h_N\,\underline{(M_1,M_2)}$$

本震発生確率　　緯度経度　距離・時間　マグニチュード

- ➢ $\mathrm{logit}(p) = \log\{p/(1-p)\}$: ロジット関数
- ➢ $P_N(X,Y,D,T,M_1,M_2)$: 地震群の後続で本震が起こる確率
- ➢ f_N: 薄板スプライン関数、g_N, h_N : B-スプライン関数
- ➢ 説明変数、交互作用の組合せ、スプライン関数は、UBRE(Unbiased Risk Estimator) を用いて選択した。

Step 4. 訓練データによる学習

- ロジスティック回帰モデル（一般化加法モデル）

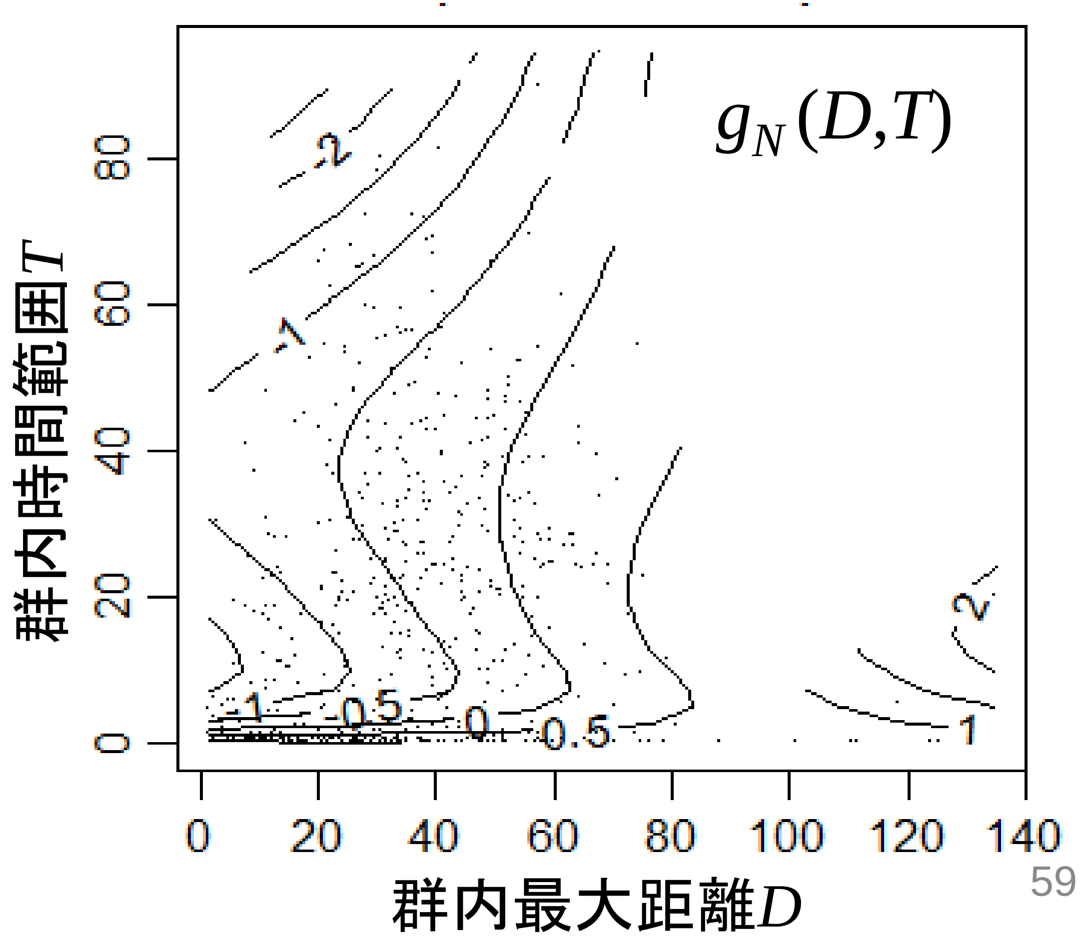
$$\mathrm{logit}(\underline{P_N(X,Y,D,T,M_1,M_2)}) = \underline{f_N(X,Y)} + \underline{g_N(D,T)} + \underline{h_N(M_1,M_2)}$$

本震発生確率　　Lon, Lat　　Dist･Time　　Mag.

- 推定結果（N=5のとき）

➢ 地震群の時間範囲がとても短い（5日以内）ときに、本震発生確率は非常に高くなる。

➢ 地震群の最大距離が長いほど本震発生確率は高くなる。（ただし時間範囲が非常に短いときは逆転している。）



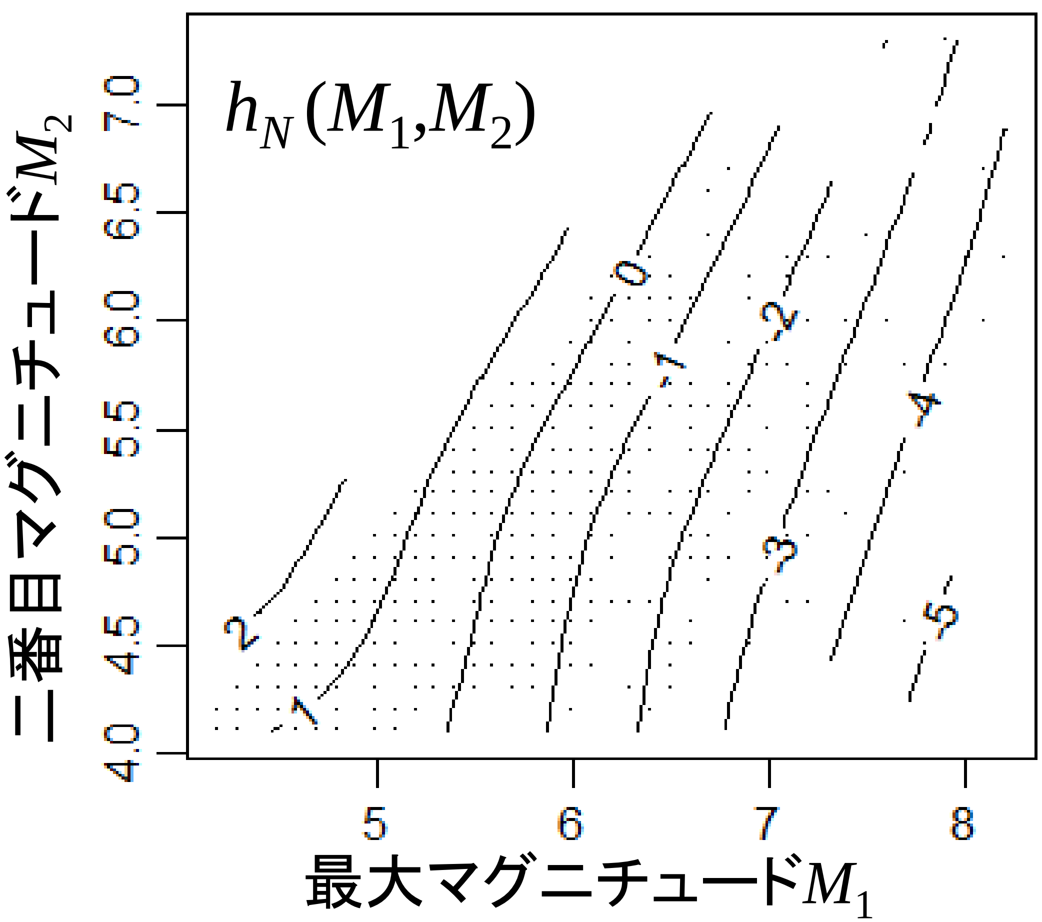
Step 4. 訓練データによる学習

- ロジスティック回帰モデル（一般化加法モデル）

$$\mathrm{logit}(P_N(X,Y,D,T,M_1,M_2)) = f_N(X,Y) + g_N(D,T) + h_N(M_1,M_2)$$

本震発生確率　緯度経度　距離・時間　マグニチュード

- 推定結果（N=5のとき）



➢ 最大マグニチュードが小さいほど本震発生確率は高い。（本震マグニチュードがそれまでの最大マグニチュードを超えなければならないのが主な理由）

➢ 二番目のマグニチュードは最大マグニチュードに近いほど本震発生確率が高い。

EXPLORING MAGNITUDE FORECASTS OF THE NEXT EARTHQUAKE

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FORMLATION OF THE ISSUES

$$P\{an\ event\ in\,a\ bin\ [t,t+\Delta t]\times[x,x+\Delta x]\times[y,y+\Delta y]\times[M,M+\Delta M]\mid H_t,\ F_t\}$$

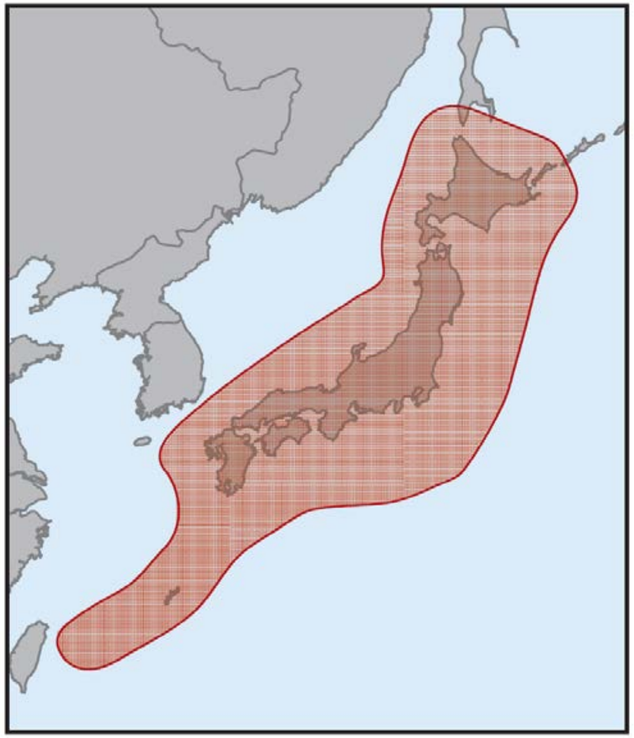
$$\approx \lambda(t,x,y,M \mid H_t, F_t)\ \Delta t\ \Delta x\ \Delta y\ \Delta M$$

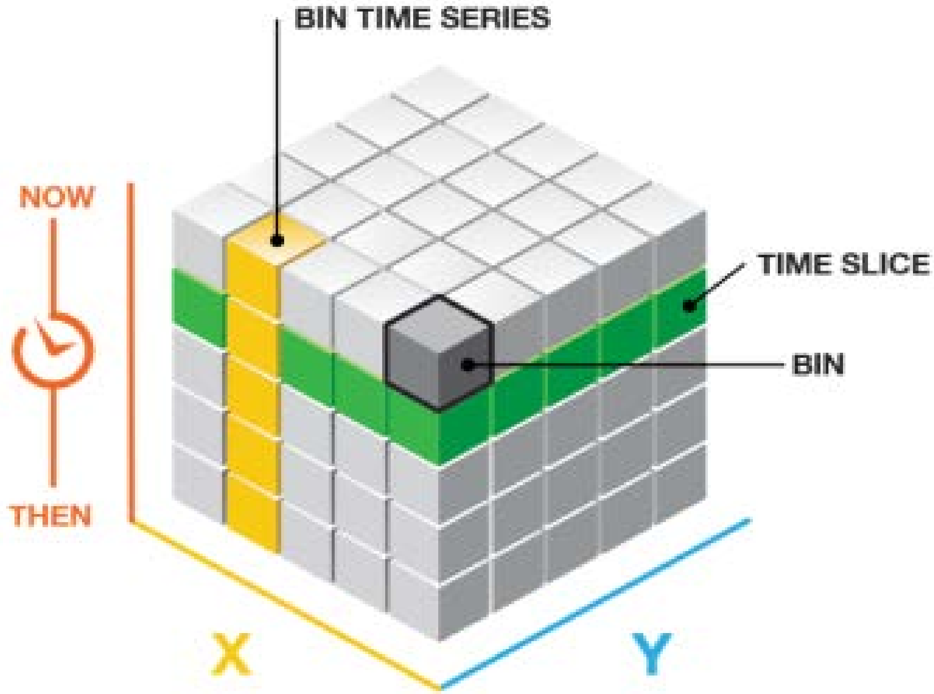
Conditional intensity function

Time ***t*****,** a ***location*** **(*x, y*),** and a ***magnitude*** *M*,

History of occurrence records: $H_t = \{\ (t_j, x_j, y_j, M_j);\ t_j < t\ \}$

History of other exogenous records: F_t



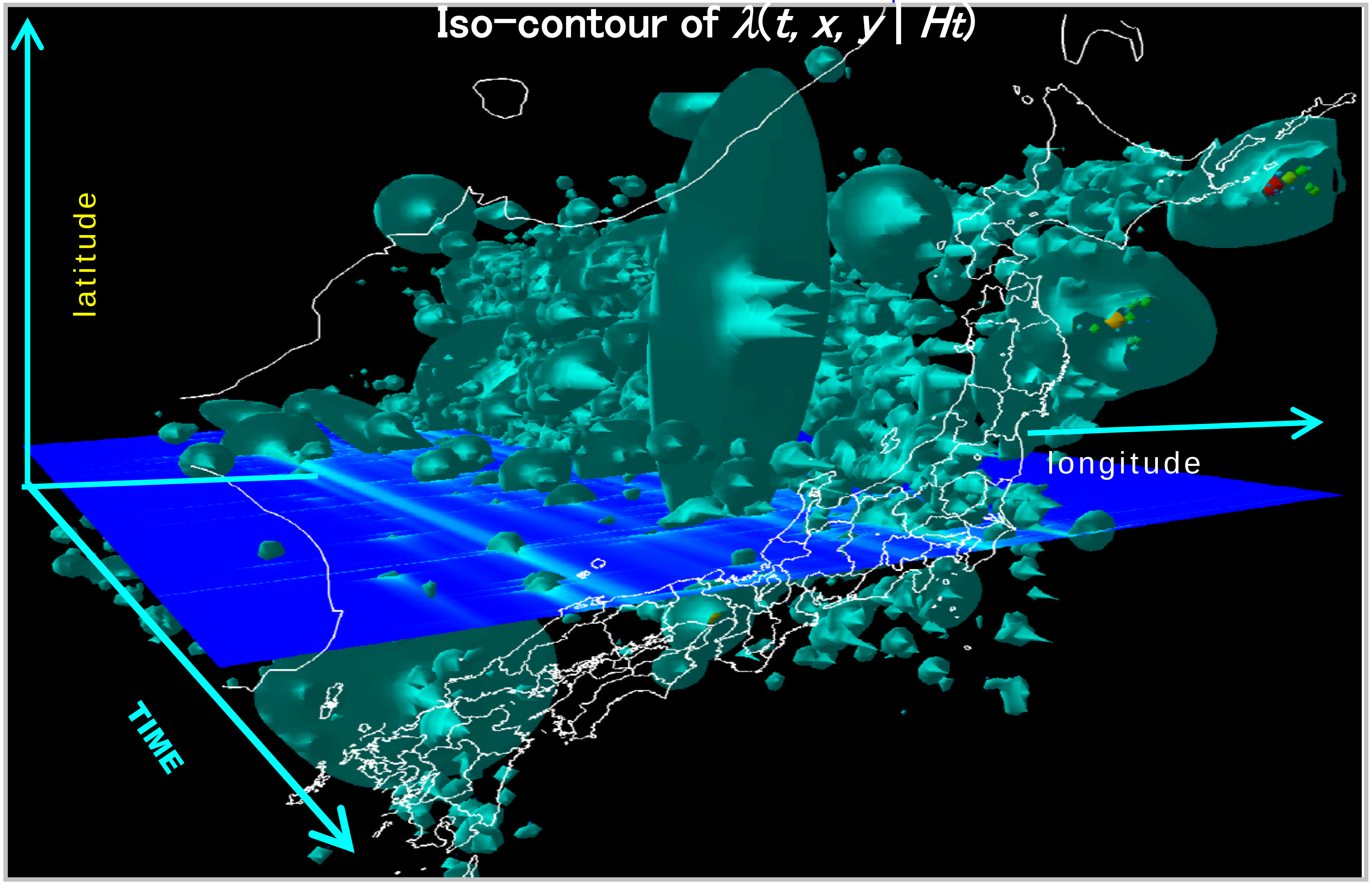


FORMLATION OF THE ISSUES

Separablity: $\lambda(t, x, y, M \mid H_t, F_t) \approx \lambda(t, x, y \mid H_t, F_t)\, \gamma(M \mid t, x, y, H_t, F_t)$

1st-term: Seismicity

$$\lambda(t, x, y \mid H_t, F_t) = \mu(x,y) + \sum_{\{j:\, t_j < t\}} \frac{K(x_j, y_j)}{(t - t_j + c)^p} \left[\frac{(x-\bar{x}_j, y-\bar{y}_j) S_j^{-1} \begin{pmatrix} x-\bar{x}_j \\ y-\bar{y}_j \end{pmatrix}}{e^{\alpha(M_j - M_c)}} + d \right]^{-q}$$



FORMLATION OF THE ISSUES

Separablity: $\lambda(t, x, y, M \mid H_t, F_t) \approx \lambda(t, x, y \mid H_t, F_t)\, \gamma(M \mid t, x, y, H_t, F_t)$

2nd term: $\gamma(M \mid t, x, y, H_t, F_t)\, dM = P\left(M < Magnitude \le M + dM \mid t, x, y, H_t, F_t\right)$

Our task:

$\gamma(M \mid t, x, y, H_t, F_t)$ ⟺ $\gamma_0(M) = 10^{a - b(M - M_c)}$

Challenging models

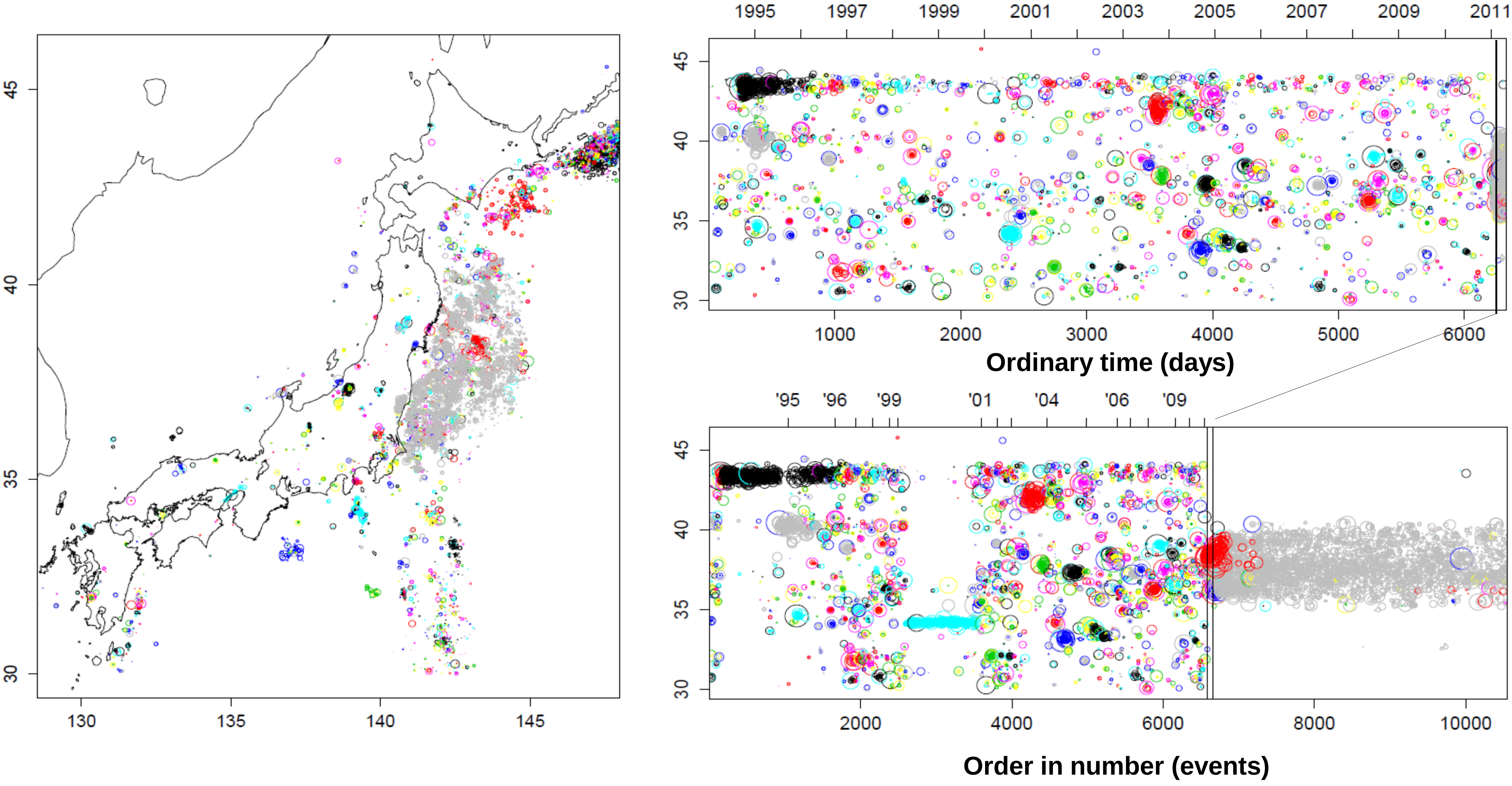
Gutenberg-Richter Law (b = 0.9) as a **reference** model

Evaluate:

Probability and information gains relative to the reference model.

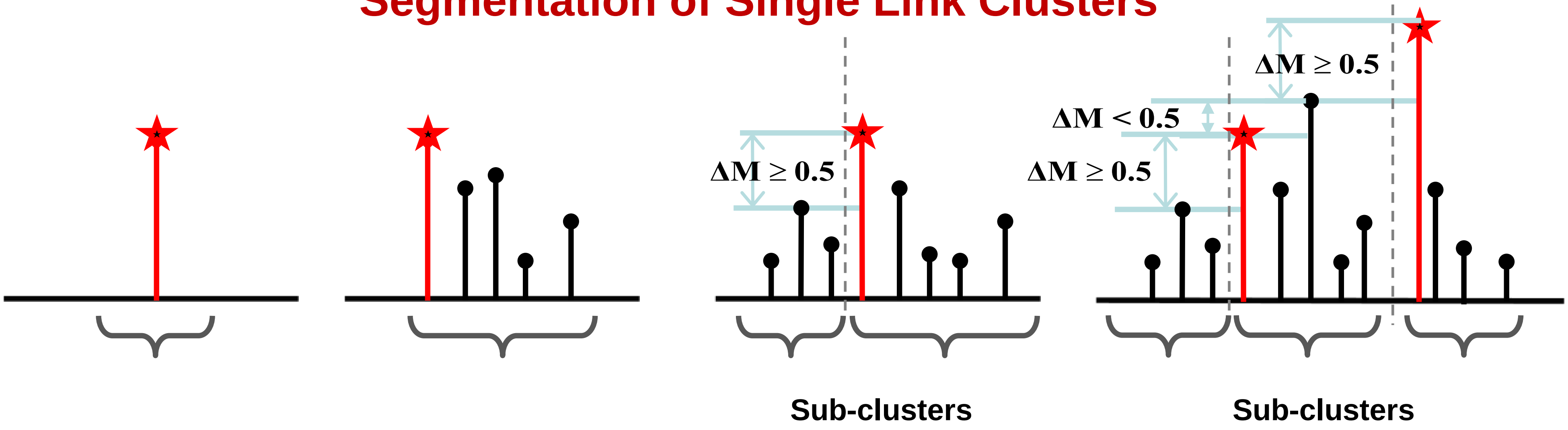
Ogata et al. (2018): CSEP Special Issue in ***Seism. Rev. Lett.***

1) **Location dependent *b*-values.**
2) **Reciprocal of *b*-value (Moving average in time and space)**
3) **Magnitude frequency distributions based on SLC foreshock discrimination**



45
40
35
30
130
135
140
145
1995
1997
1999
2001
2003
2005
2007
2009
2011
1000
2000
3000
4000
5000
6000
Ordinary time (days)
'95
'96
'99
'01
'04
'06
'09
2000
4000
6000
8000
10000
Order in number (events)

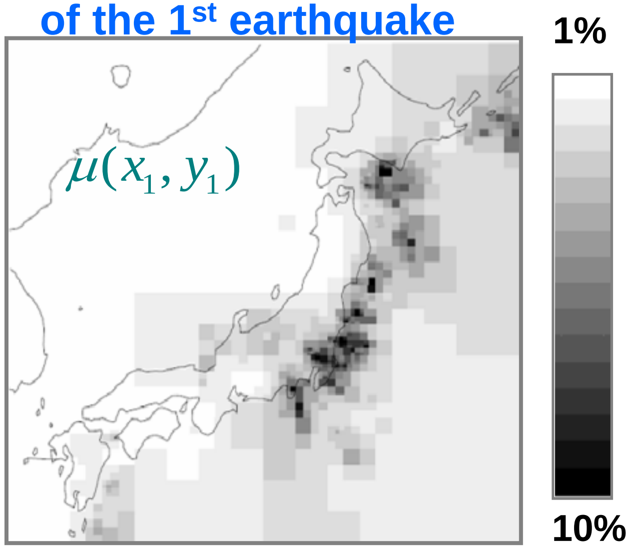
Segmentation of Single Link Clusters

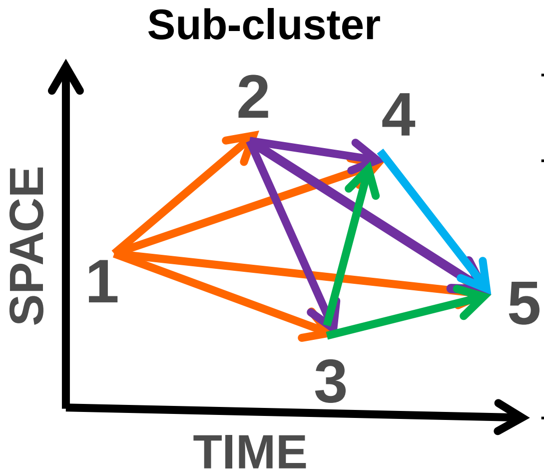


$p_{c|n}$ = Probability of $M_n \geqq M_{max} + 0.5$ at the n-th earthquake of subgroup in c

$$\ln \frac{1 - p_{c|n}}{p_{c|n}} = \ln \frac{1 - \mu(x_1, y_1)}{\mu(x_1, y_1)} + \frac{1}{\#\{i < j\}} \sum_{i<j<n} \left[a_1 + \sum_{k=1}^{3} b_k \gamma_{i,j}{}^{k} + \sum_{k=1}^{3} c_k \rho_{i,j}{}^{k} + \sum_{k=1}^{3} d_k \tau_{i,j}{}^{k} \right]$$

Foreshock probability of the 1st earthquake





Coeff. from Ogata, Utsu and Katsura, 1996, *GJI*

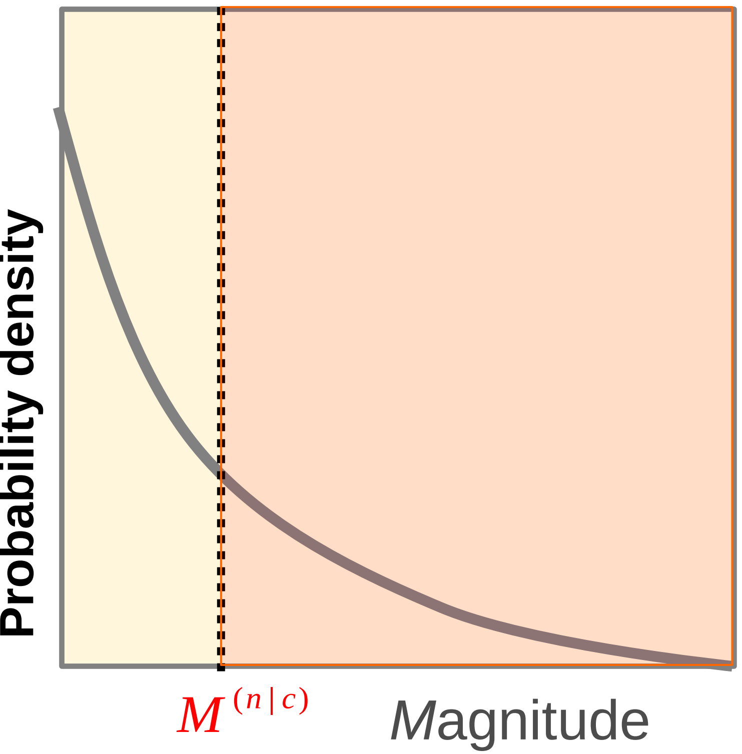
k	a_k	b_k	c_k	d_k
1	8.018	−33.25	−1.490	−10.92
2		62.77	2.805	295.09
3		−37.66	−2.190	−1161.50

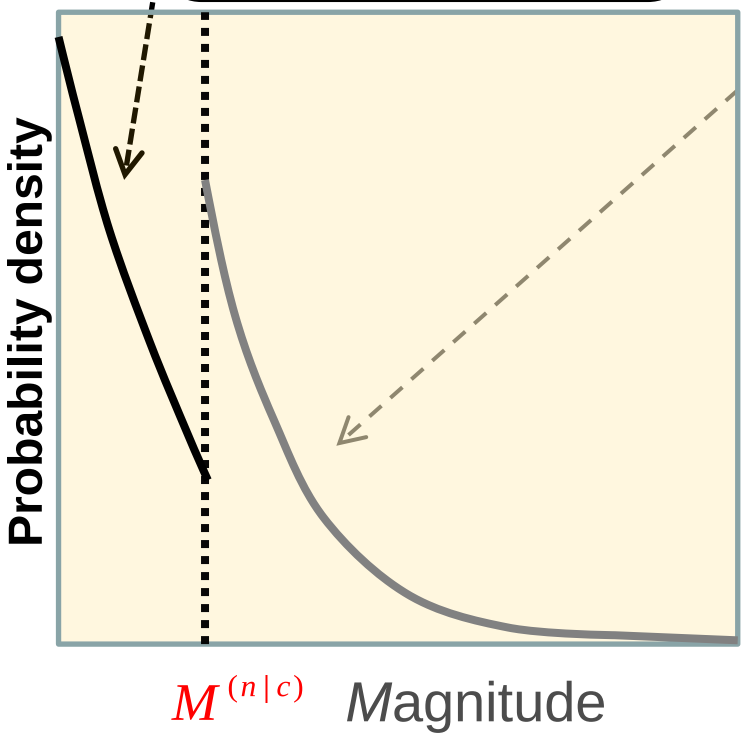
Leaping Magnitude Threshold : $M^{(n|c)} = \max\{M_k;\ k=1,\cdots,n \mid \text{in cluster } c\} + 0.5$

Probability of M≧Mmax+0.5 of the next magnitude; $p_{n|c} = P\{M_{n+1} > M^{(n|c)} \mid in\ c\}$

If $(t_{n+1}, x_{n+1}, y_{n+1})$ is connected to c,

$$\Psi(M \mid M_1,\cdots,M_n) = (1 - p_{n|c})\frac{1_{(M_c,\,M^{(n|c)})}(M)\cdot 10^{-bM}}{\int_{M_c}^{M^{(n|c)}} 10^{-bM}\,dM} + p_{n|c}\frac{1_{(M^{(n|c)},\,\infty)}\cdot 10^{-bM}}{\int_{M^{(n|c)}}^{\infty} 10^{-bM}\,dM}$$





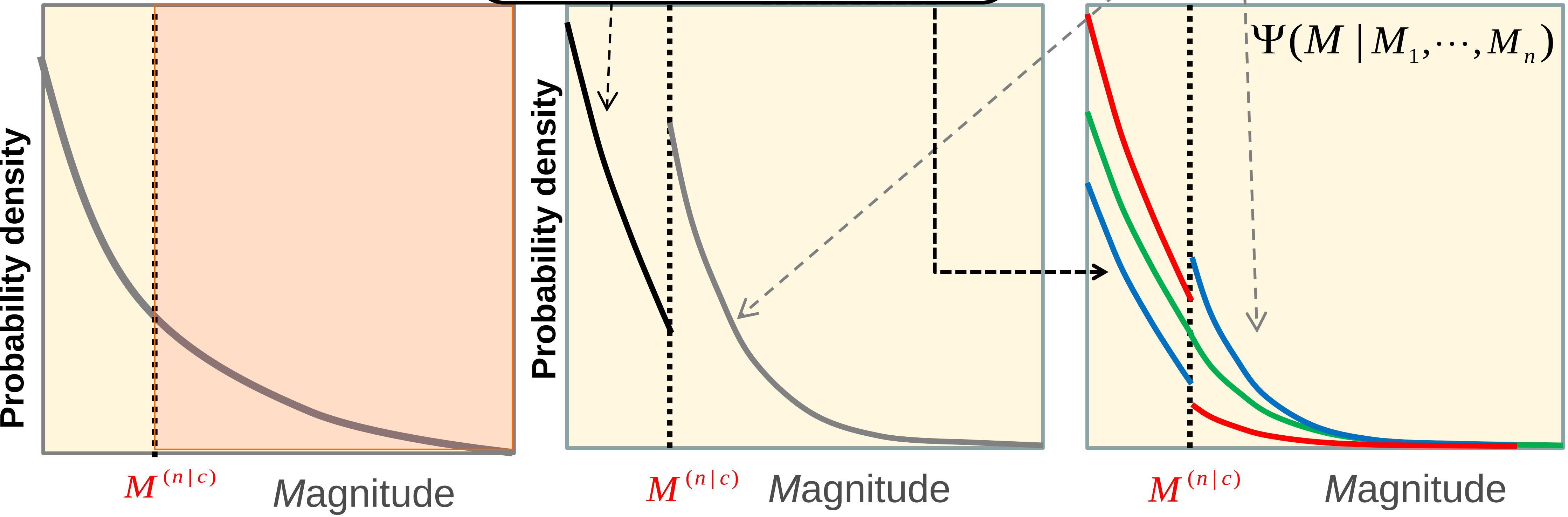
$$\Psi_{GR}(M) = \frac{1_{(M_c,\infty)}(M)\cdot 10^{-b\,M}}{\int_{M_c}^{\infty} 10^{-b\,M}\,dM}$$

Leaping Magnitude Threshold : $M^{(n|c)} = \max\{M_k;\ k=1,\cdots,n\,|\,\text{in cluster } c\} + 0.5$

Probability of M≧Mmax+0.5 of the next magnitude; $p_{n|c} = P\{M_{n+1} > M^{(n|c)}\,|\,in\ c\}$

If $(t_{n+1}, x_{n+1}, y_{n+1})$ is connected to c,

$$\Psi(M\,|\,M_1,\cdots,M_n) = (1-p_{n|c})\frac{1_{(M_c,\,M^{(n|c)})}(M)\cdot 10^{-bM}}{\int_{M_c}^{M^{(n|c)}} 10^{-bM}\,dM} + p_{n|c}\frac{1_{(M^{(n|c)},\,\infty)}\cdot 10^{-bM}}{\int_{M^{(n|c)}}^{\infty} 10^{-bM}\,dM}$$

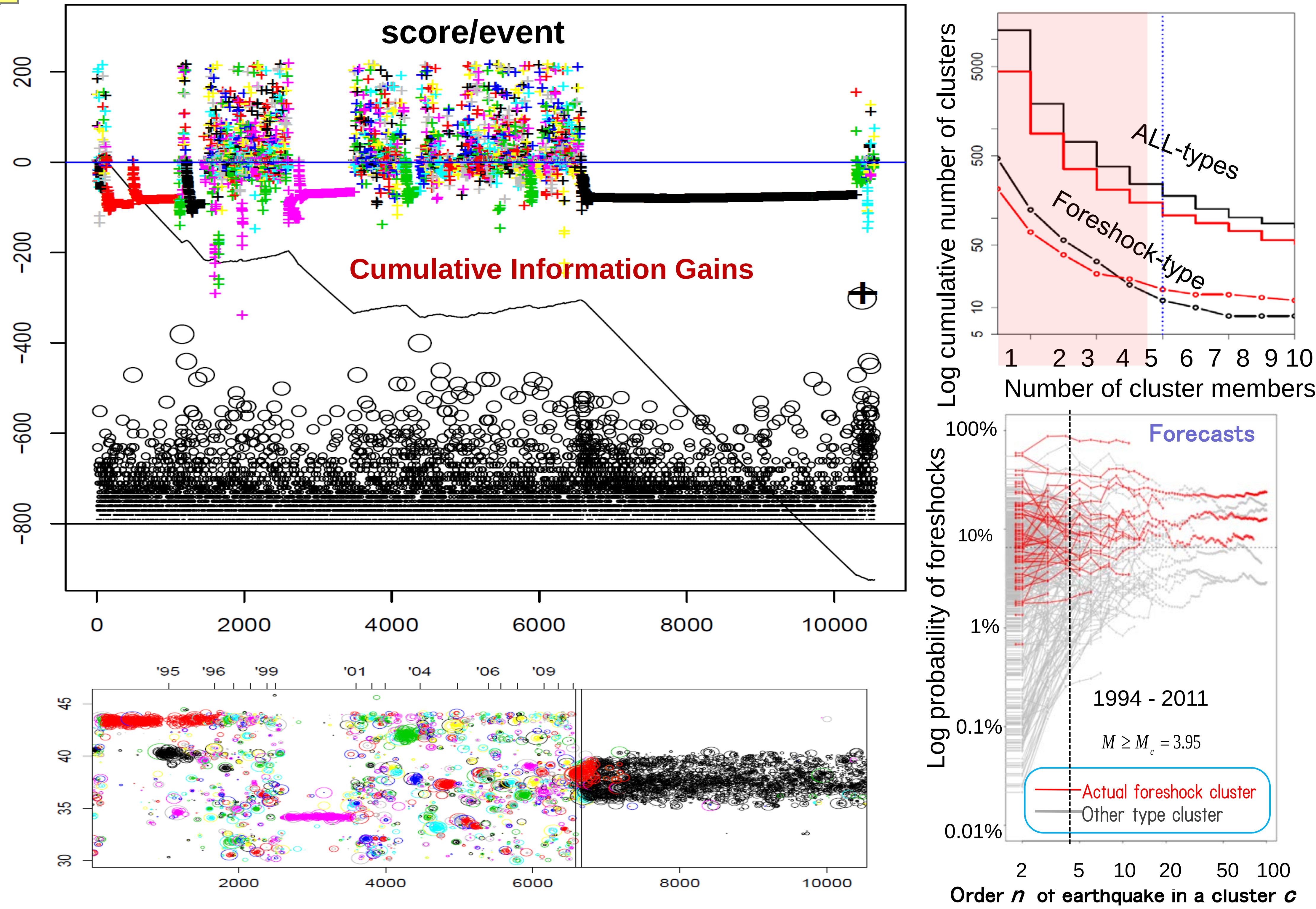


Compare with the reference G-R model

$$\Psi(M) = 1_{(M_c,\infty)}(M)\cdot 10^{-bM} \Big/ \int_{M_c}^{\infty} 10^{-bM}\,dM$$

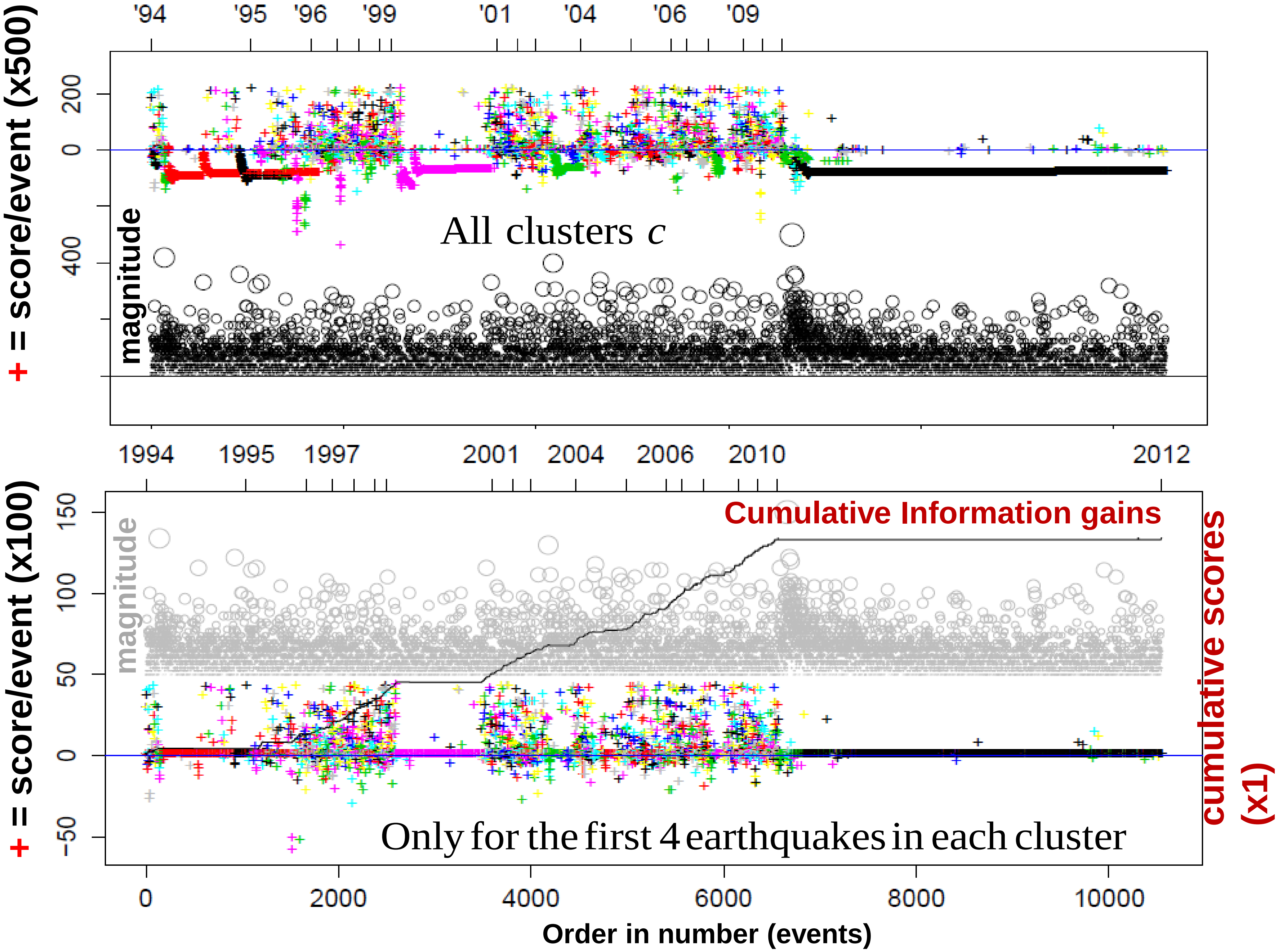
log likelihood-ratio = information gain: $\log L/L_0 = \sum_c \sum_{n=1}^{\#c} \log \frac{\Psi_c(M_{n+1}\,|\,M_c^{(n)})}{\Psi_c(M_{n+1})}$

Information gain / earthquake



score/event
Cumulative Information Gains
200
0
-200
-400
-600
-800
0
2000
4000
6000
8000
10000
'95
'96
'99
'01
'04
'06
'09
45
40
35
30
Log cumulative number of clusters
5000
500
50
10
5
ALL-types
Foreshock-type
1 2 3 4 5 6 7 8 9 10
Number of cluster members
Log probability of foreshocks
100%
10%
1%
0.1%
0.01%
Forecasts
1994 - 2011
$M \geq M_c = 3.95$
Actual foreshock cluster
Other type cluster
2 5 10 20 50 100
Order n of earthquake in a cluster c

Information gain scores; All Japan 1994 – 2011, $M \geq 4$



Summary and suggestions

(1) I have shown useful statistical magnitude forecast of next earthquake, which depends on history of some seismicity characteristics such as space-time clustering concentration and magnitude increments.

(2) These were confirmed by comparison of the forecasts with the independent G-R distribution as a reference using log-likelihood (= information gain).

(3) We can carry out testing by topical magnitude score of log conditional likelihood for diagnostic purpose to improve the proposed model of magnitude distribution.

Foreshock features relative to the mainshock

Characteristics of foreshocks in time, space and magnitude can be simulated by the **ETAS model** and the G−R law

(e.g. Helmstetter, et al. , 2003, JGR; Felzer et al., 2004, BSSA; Marzocchi and Zhuang, GRL 2011).

QUESTIONS on the ETAS simulations

➔

1. Are these characteristics quantitatively similar to those in real earthquake catalog?

2. Is the concentrating degree of foreshocks independent of the mainshock size?

3. Is the b-value of foreshocks larger than that of aftershocks?

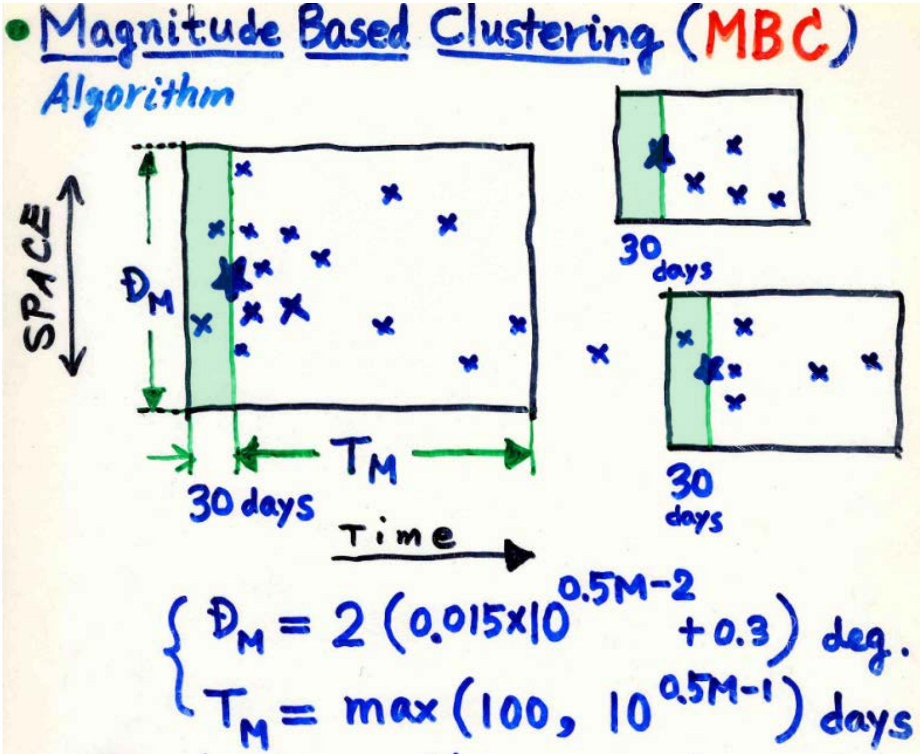
4. Can the foreshock-based forecasts improve prediction by ETAS models?

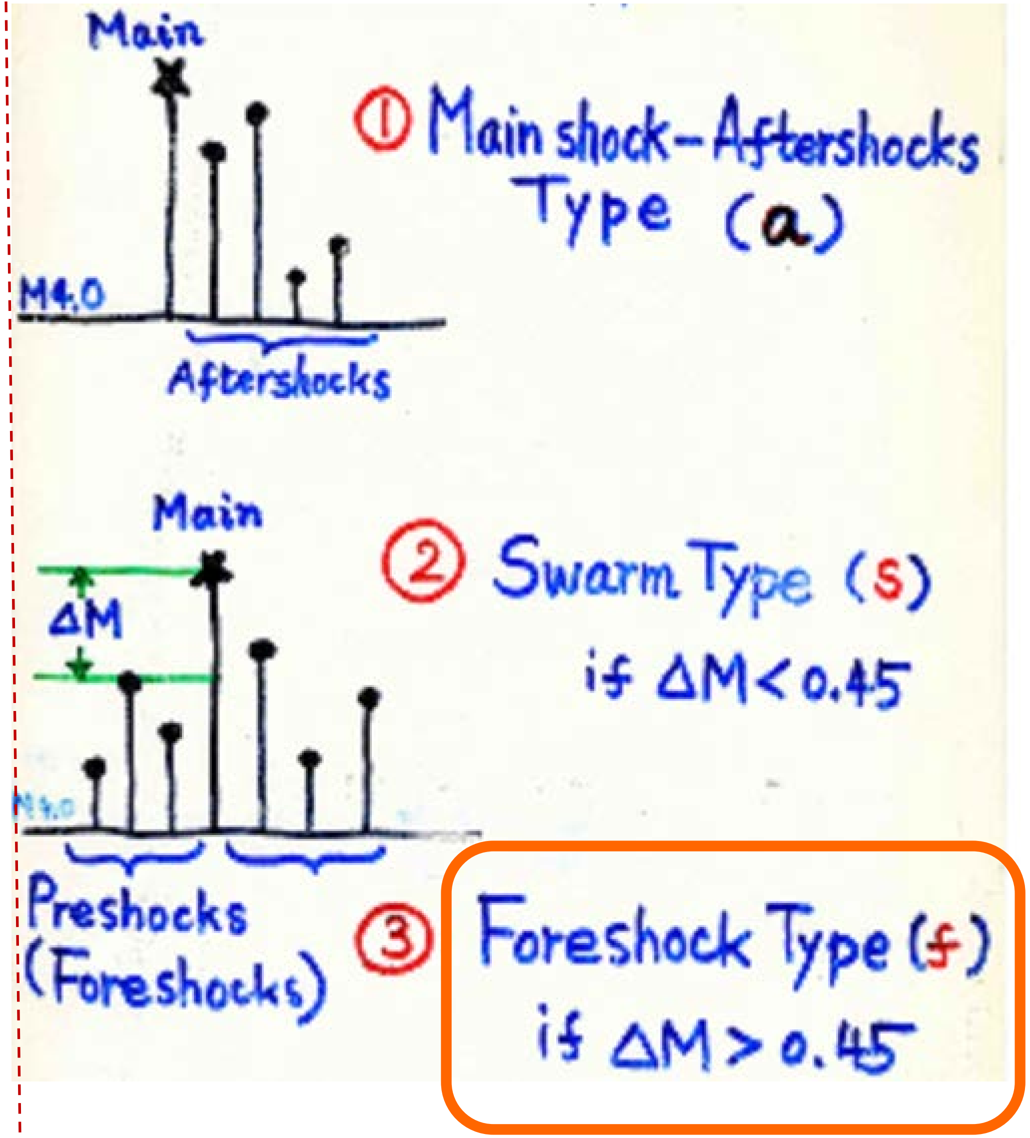
Clustering

Cluster types

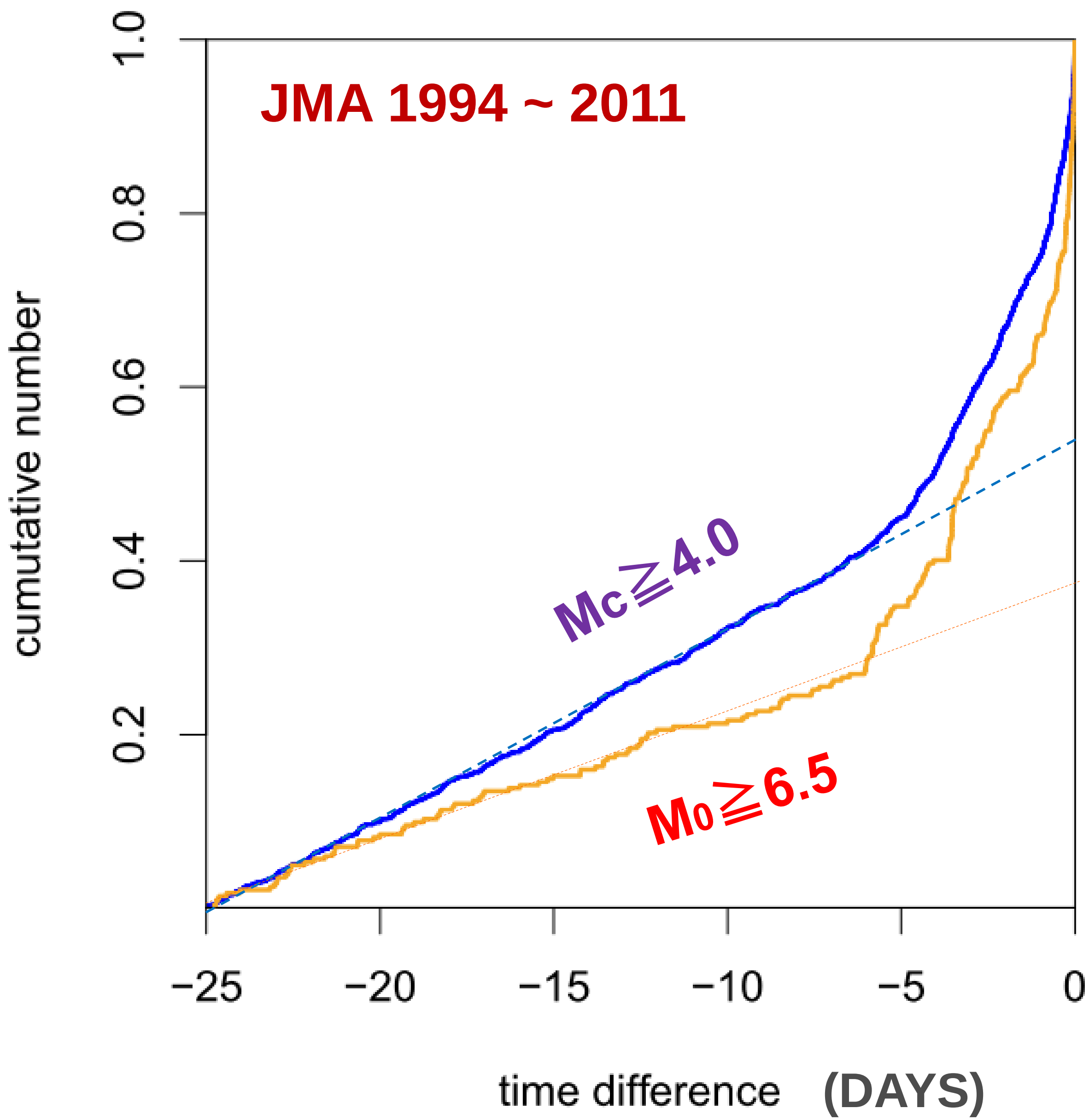
Magnitude-based methods

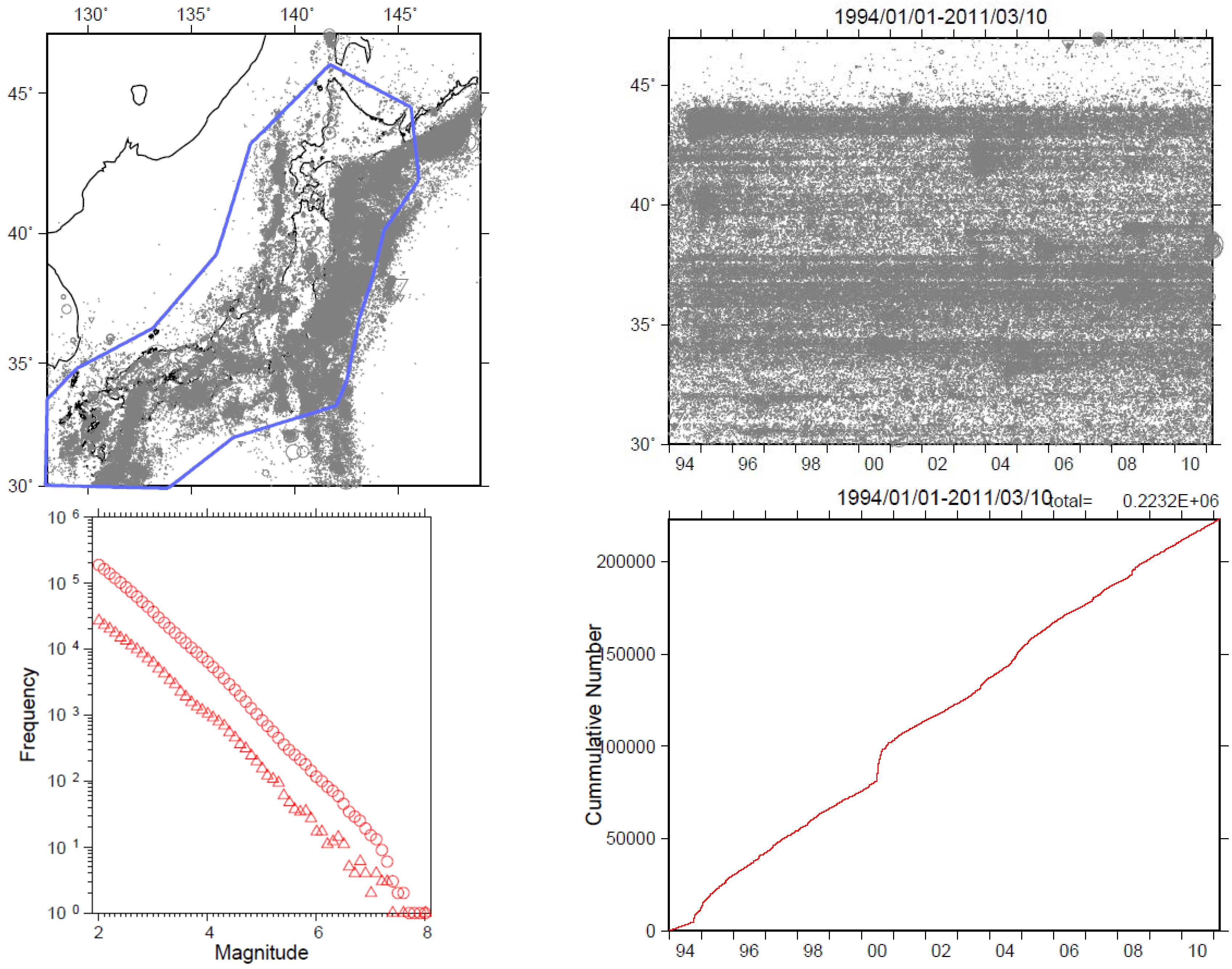
(Utsu,1969,*Geophys.Bull.Hokkaido U.*)





Cumulative number of stacked foreshocks v.s. times to the mainshock




130˚
135˚
140˚
145˚
45˚
40˚
35˚
30˚
1994/01/01-2011/03/10
94
96
98
00
02
04
06
08
10
1994/01/01-2011/03/10
total= 0.2232E+06
200000
150000
100000
50000
0
Cummulative Number
Frequency
10 6
10 5
10 4
10 3
10 2
10 1
10 0
2
4
6
8
Magnitude

$$\lambda_{ETAS}(t,x,y\,|\,H_t) = \mu(x,y) + \sum_{\{j;t_j<t\}} k(M_j)g(t-t_j)g(x-x_j, y-y_{ij})$$

$$= \mu \cdot \nu(x,y) + \sum_{\{j:\,t_j<t\}} \frac{K}{(t-t_j+c)^p}\left[\frac{(x-x_j)^2+(y-y_j)^2}{e^{\alpha(M_j-M_c)}}+d\right]^{-q}$$

$$\lambda(t,x,y,M\,|\,H_t) = f(M\,|\,H_t) \times \lambda_{ETAS}(t,x,y\,|\,H_t)$$

ETAS / M*JMA*(4.0): $f(M\,|\,H_t),\ M \geq 4.0$

Use the same magnitude sequence $\{M_j;\ M_j \geq 4.0\}$ as the JMA catalog.

ETAS / M*BRS*(4.0): $f(M\,|\,H_t) = f(M),\ M \geq 4.0$

Simulate by random sampling from the magnitudes data of the JMA catalog.

ETAS / M*JMA*(4.0|2.0): $f(M\,|\,H_t),\ M \geq 2.0;\ M_c = 4.0$

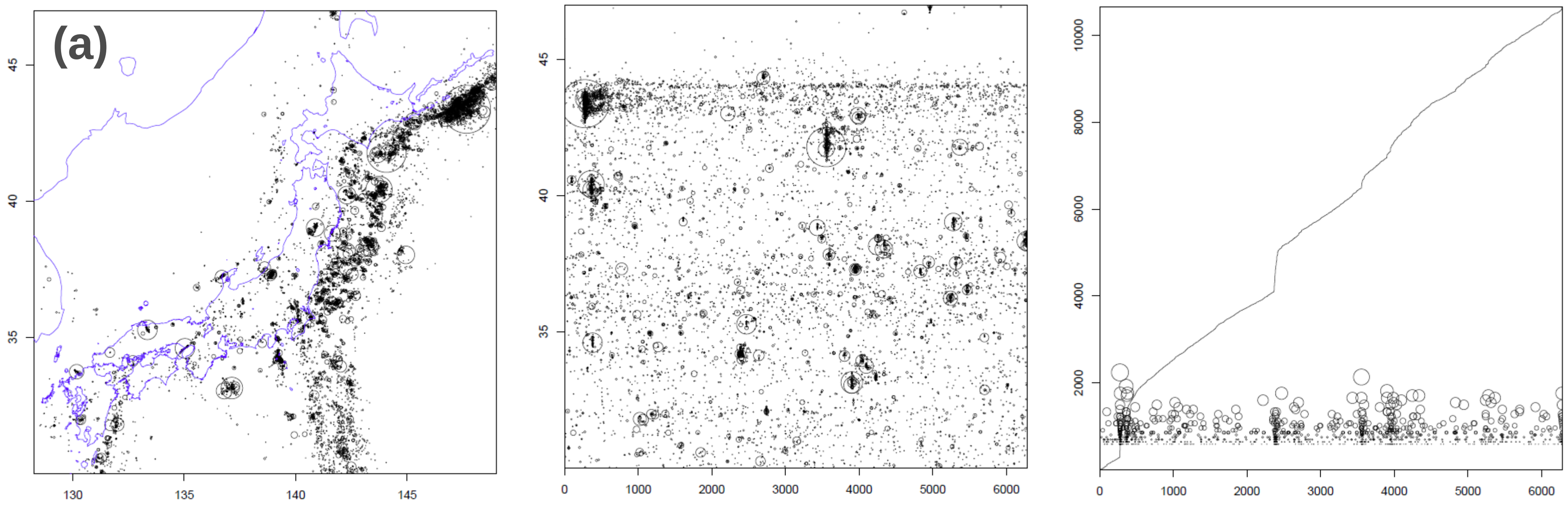
Use the same magnitude sequence $\{M_j;\ M_j \geq 2.0\}$ as the JMA catalog. Then simulate ETAS sequence $\{(t_j, x_j, y_j, M_j);\ M_j \geq 2.0\}$ and select these by cutoff magnitude $M\,4.0$

ETAS / M*BRS*(4.0|2.0): $f(M\,|\,H_t) = f(M),\ M \geq 2.0;\ M_c = 4.0$

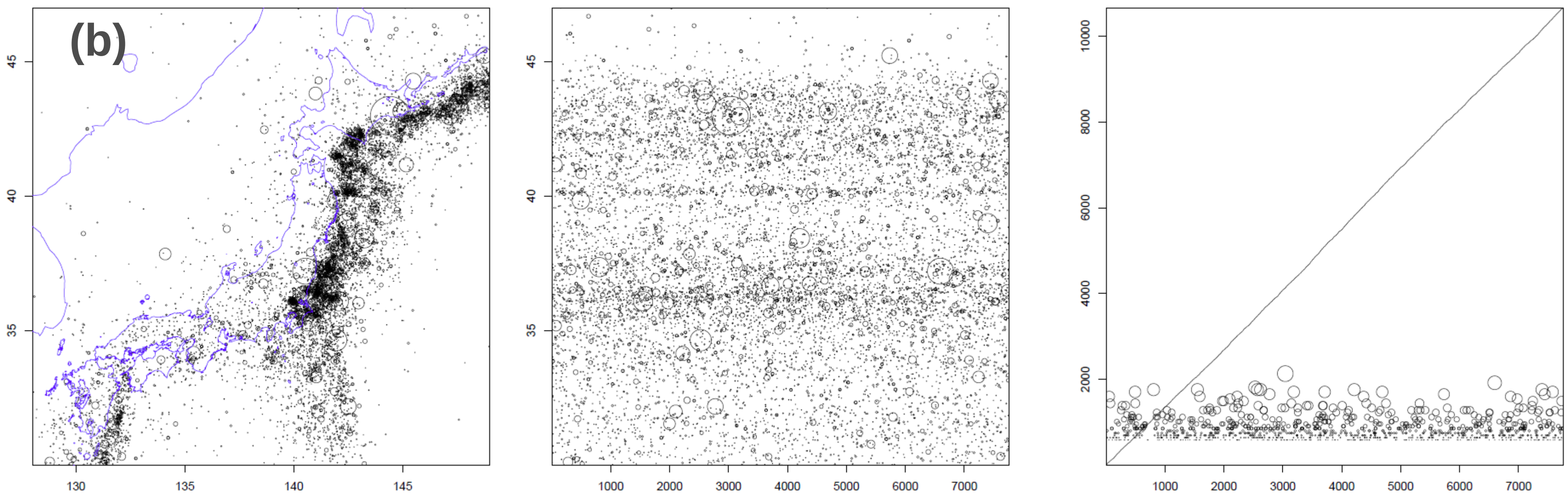
Randomly sample from the JMA magnitude sequence $\{M_j;\ M_j \geq 2.0\}$. Then simulate ETAS sequence $\{(t_j, x_j, y_j, M_j);\ M_j \geq 2.0\}$ and select these by cutoff magnitude $M\,4.0$

Poisson / M*BRS*(4.0): $\lambda(t,x,y,M\,|\,H_t) \propto \mu(x,y) f(M),\quad M \geq 4.0$

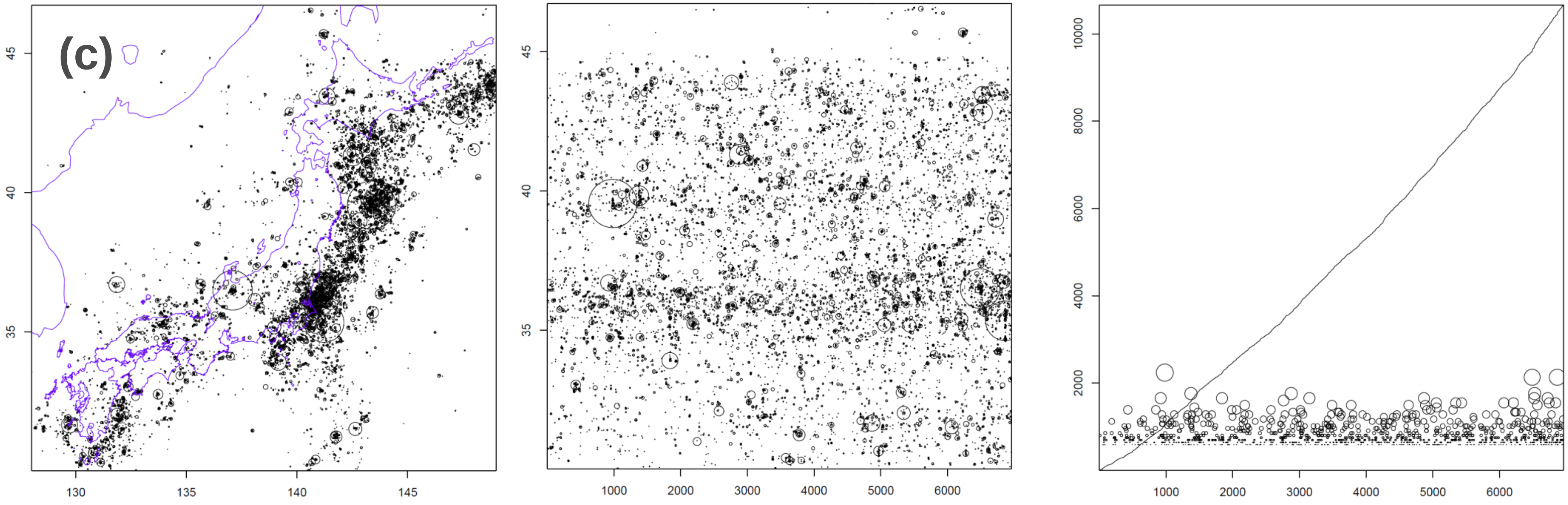
JMA catalog



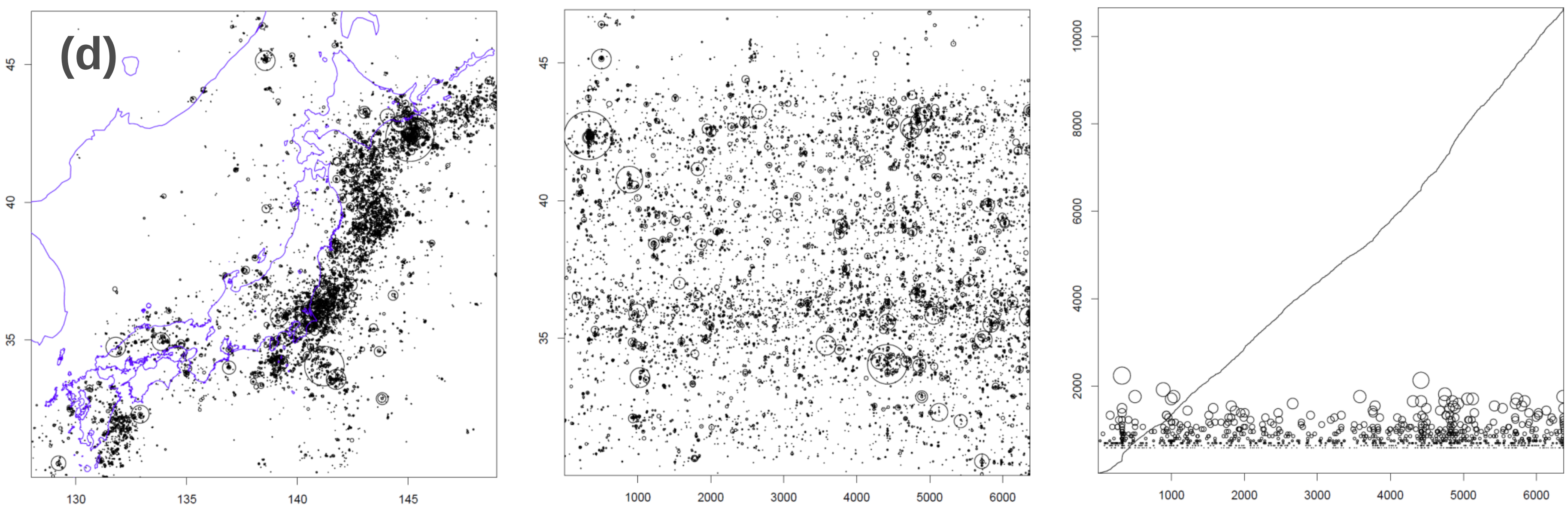
Poisson/MBRS(4.0) synthetic catalog



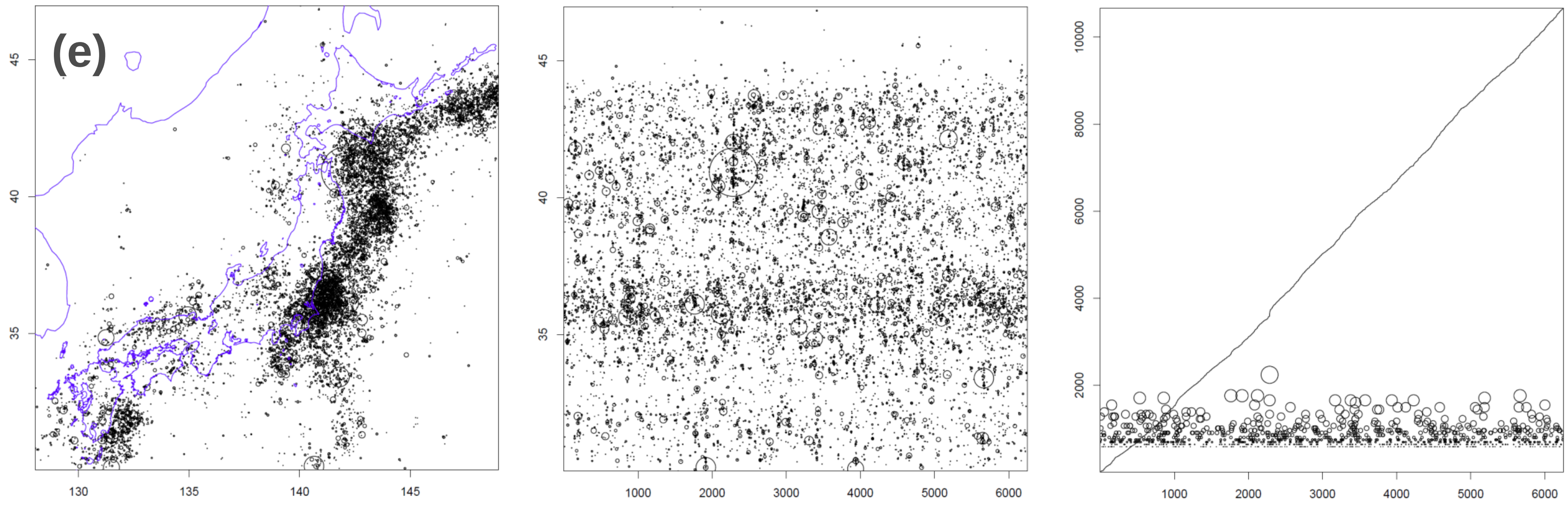
ETAS/MBRS(4.0) synthetic catalog



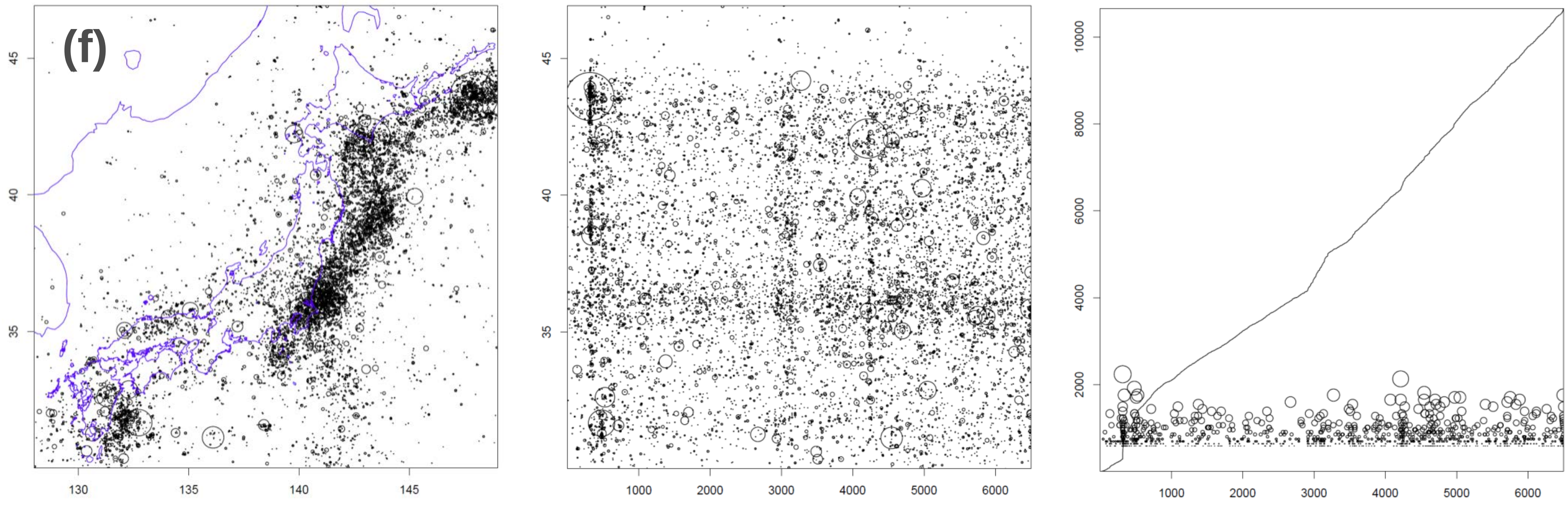
ETAS/MJMA(4.0) synthetic catalog



ETAS/MBRS(4.0|2.0) synthetic catalog

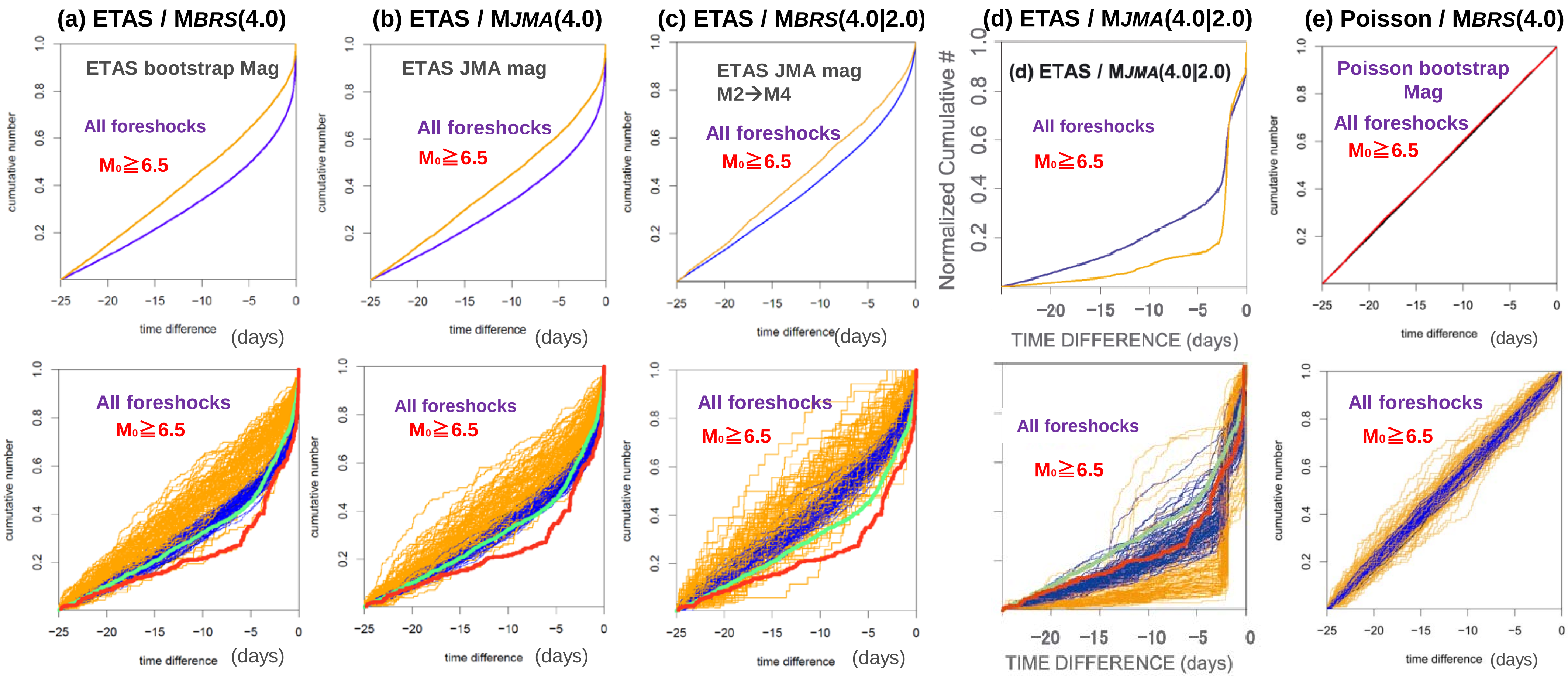


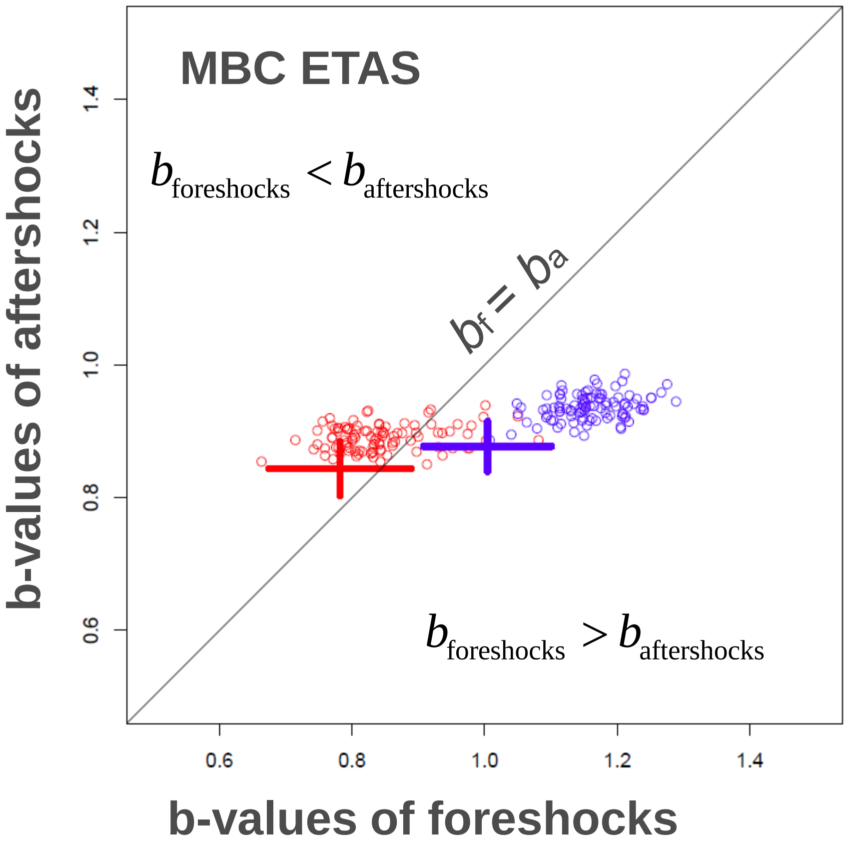
ETAS/MJMA(4.0|2.0) synthetic catalog



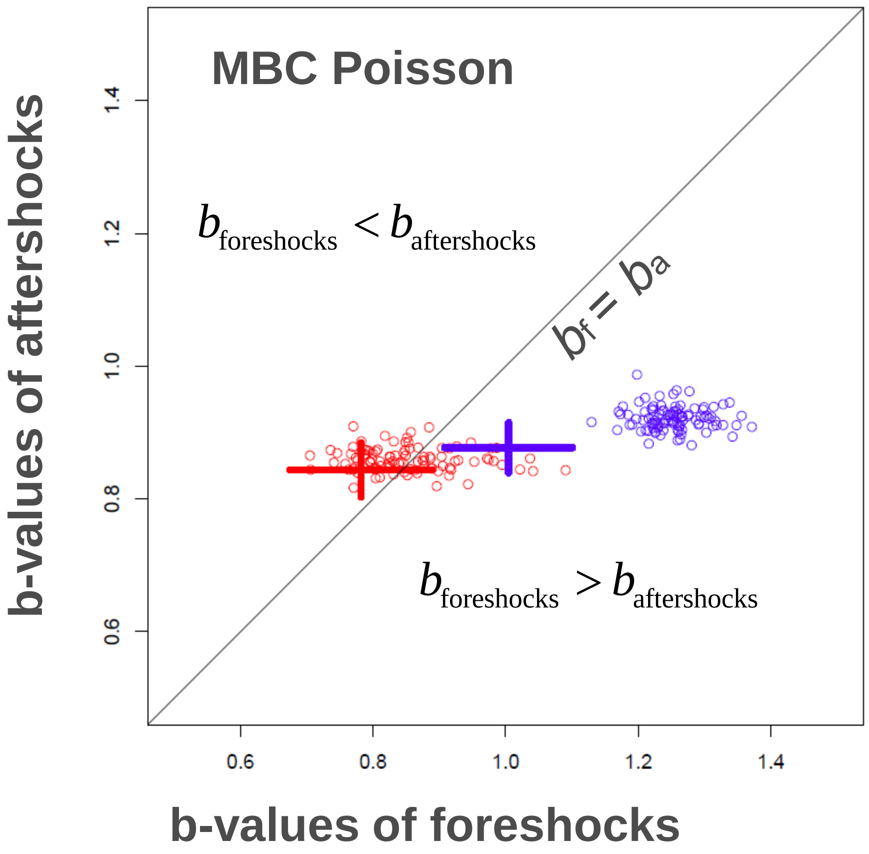
Normalized cumulative number of **stacked** foreshocks v.s. their time differences to the mainshock

Simulated experiments





MBC ETAS
$b_{\text{foreshocks}} < b_{\text{aftershocks}}$
$b_f = b_a$
$b_{\text{foreshocks}} > b_{\text{aftershocks}}$
b-values of aftershocks
b-values of foreshocks
0.6
0.8
1.0
1.2
1.4

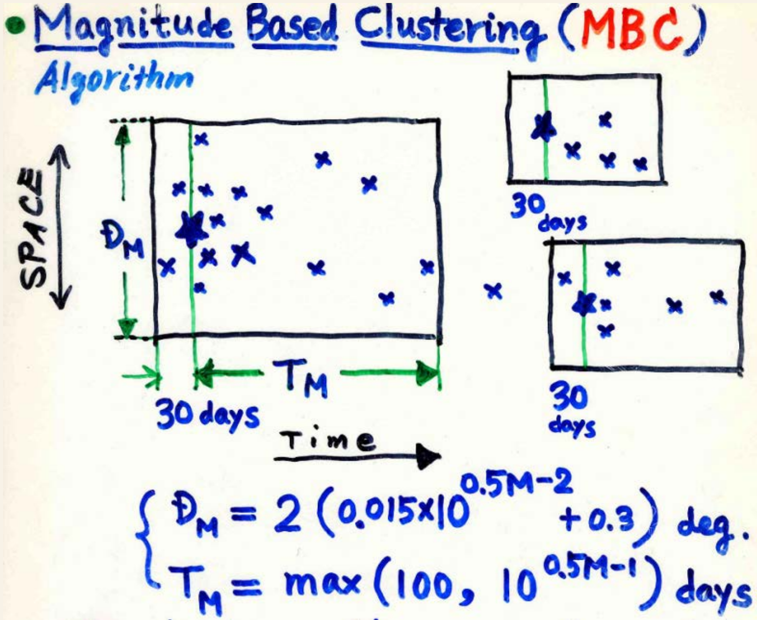


MBC Poisson
$b_{\text{foreshocks}} < b_{\text{aftershocks}}$
$b_f = b_a$
$b_{\text{foreshocks}} > b_{\text{aftershocks}}$
b-values of aftershocks
b-values of foreshocks
0.6
0.8
1.0
1.2
1.4

Clustering algorithms

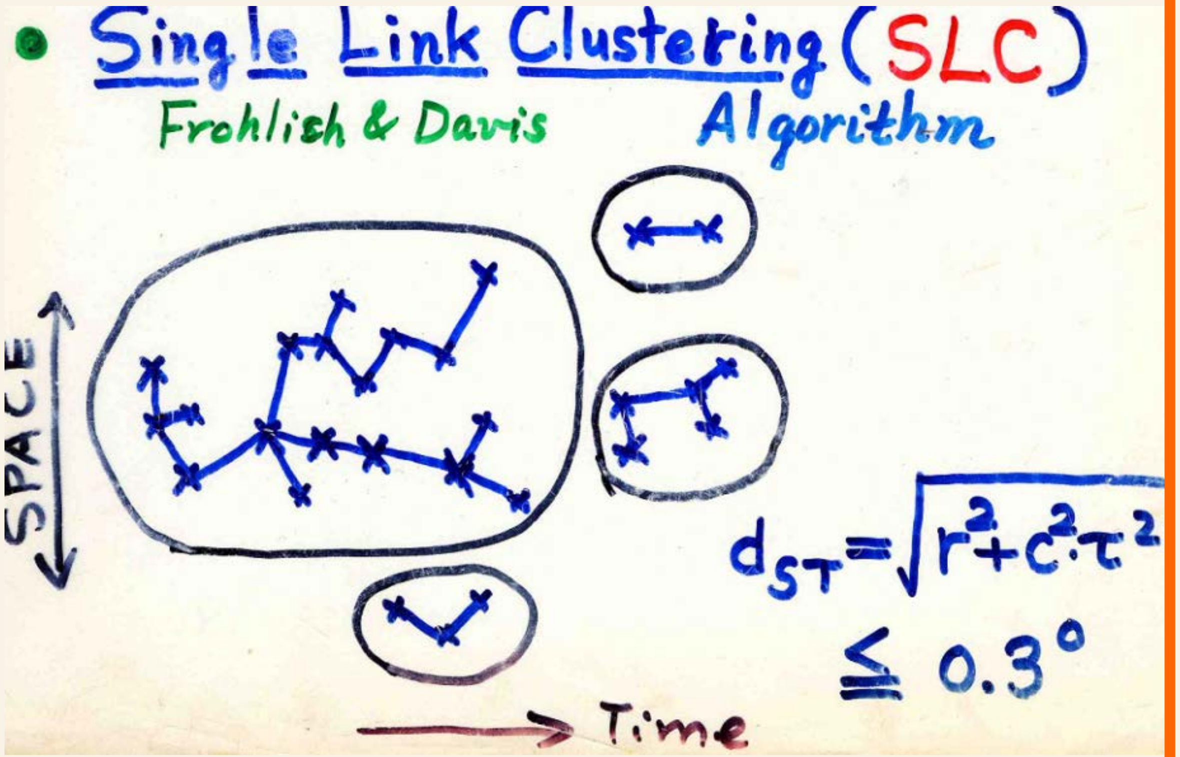
Magnitude-based

Utsu (1969, *GBHU*)



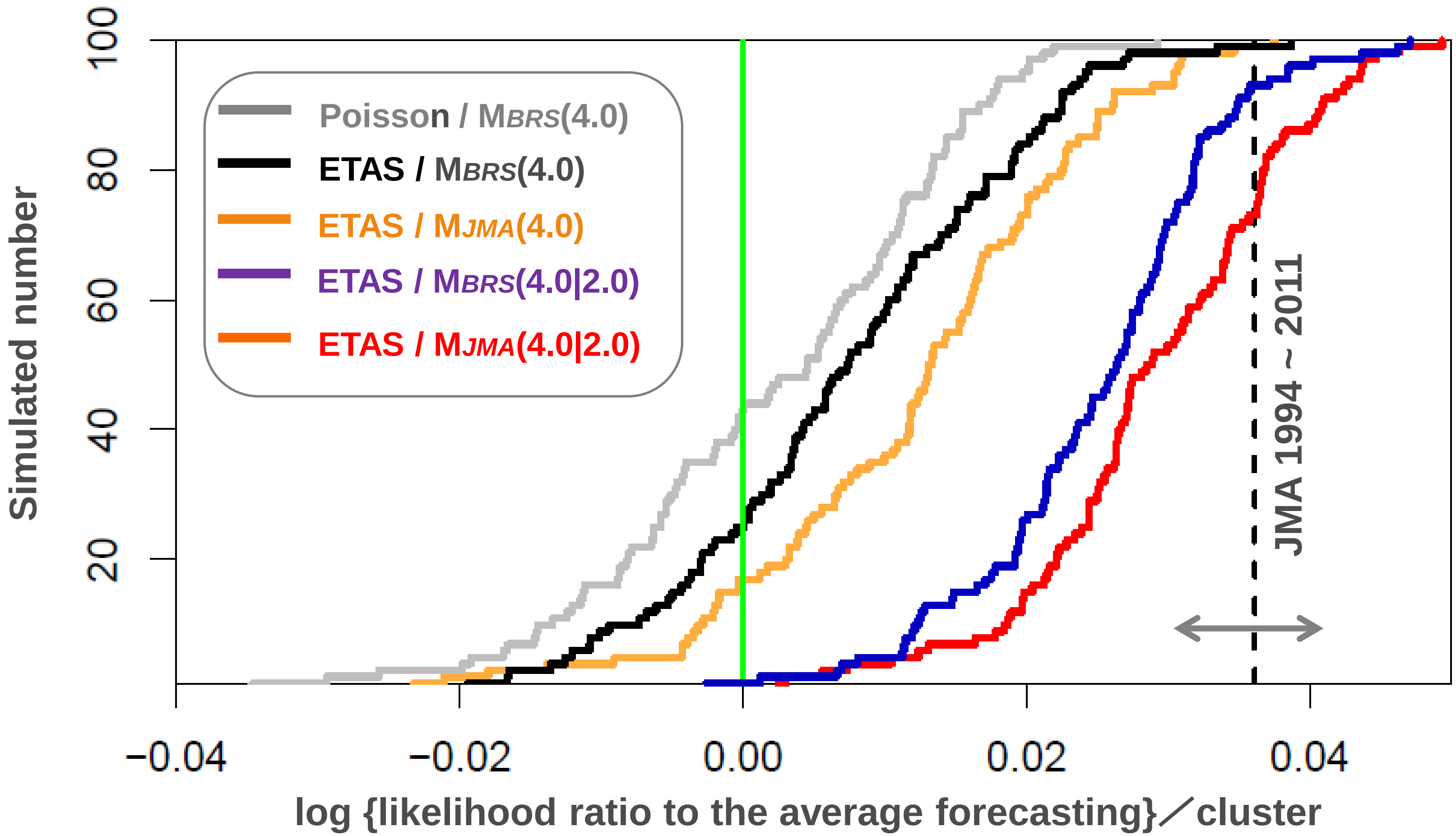
Single-Link

Frohlich & Davis (1990, *GJI*)



$$d_{ST} = \sqrt{\Delta^2{}_{space} + (c\Delta_{time})^2} \le 0.3^\circ \text{ (or 33.33km)}$$

$$c = 1^\circ\ /\ month \approx 1\ km\ /\ day$$

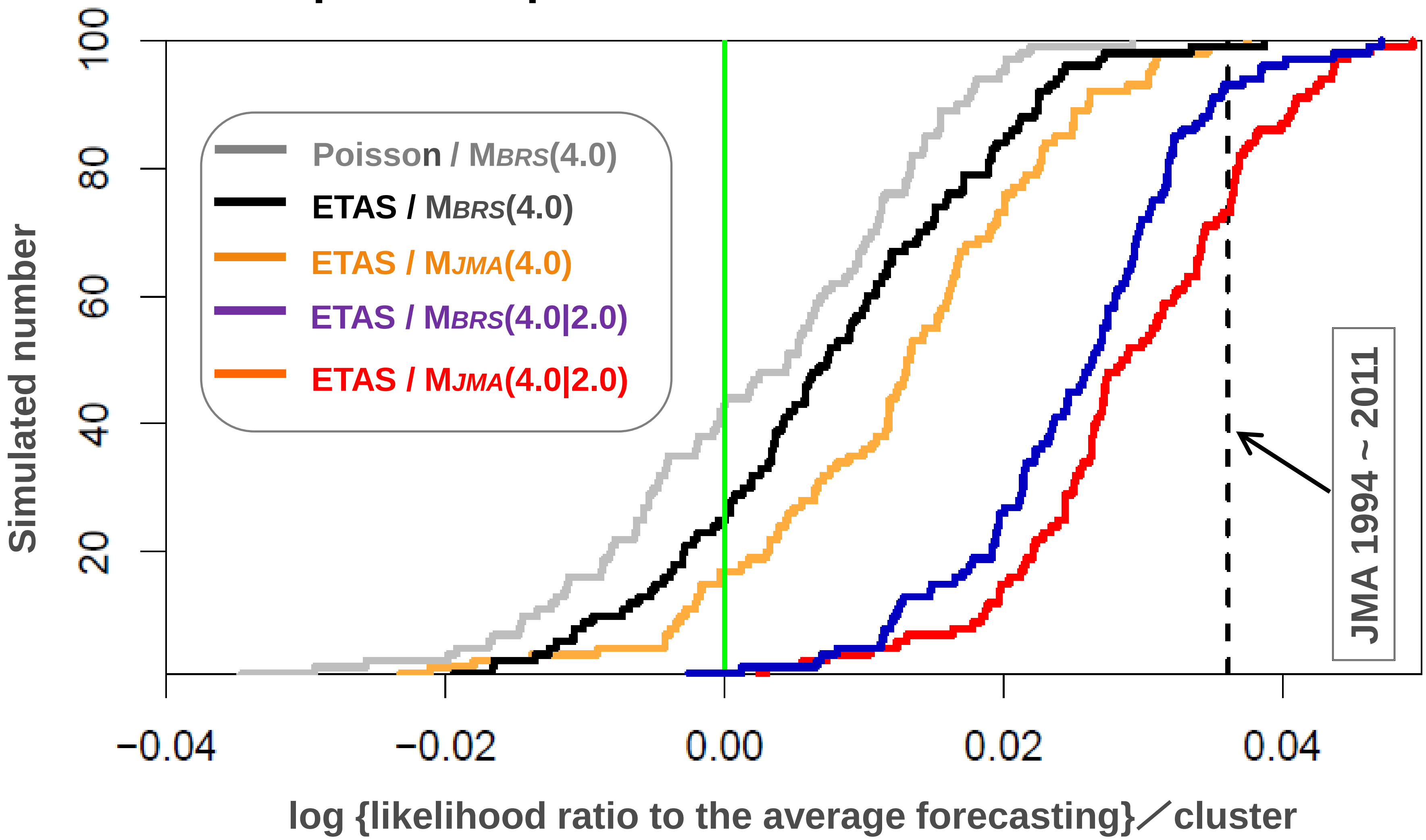


log {likelihood ratio to the average forecasting}／cluster

$$\frac{\log\{likelihood\ ratio\}}{number\ \ of\ \ clusters} = \frac{1}{\#c}\ln\left\{\frac{L_1}{L_0}\right\} = \frac{1}{\#c}\sum_{c}\left\{\eta_c \ln\frac{p_c^{\eta_c}}{\bar{p}} + (1-\eta_c)\ln\frac{1-p_c^{\eta_c}}{1-\bar{p}}\right\}$$

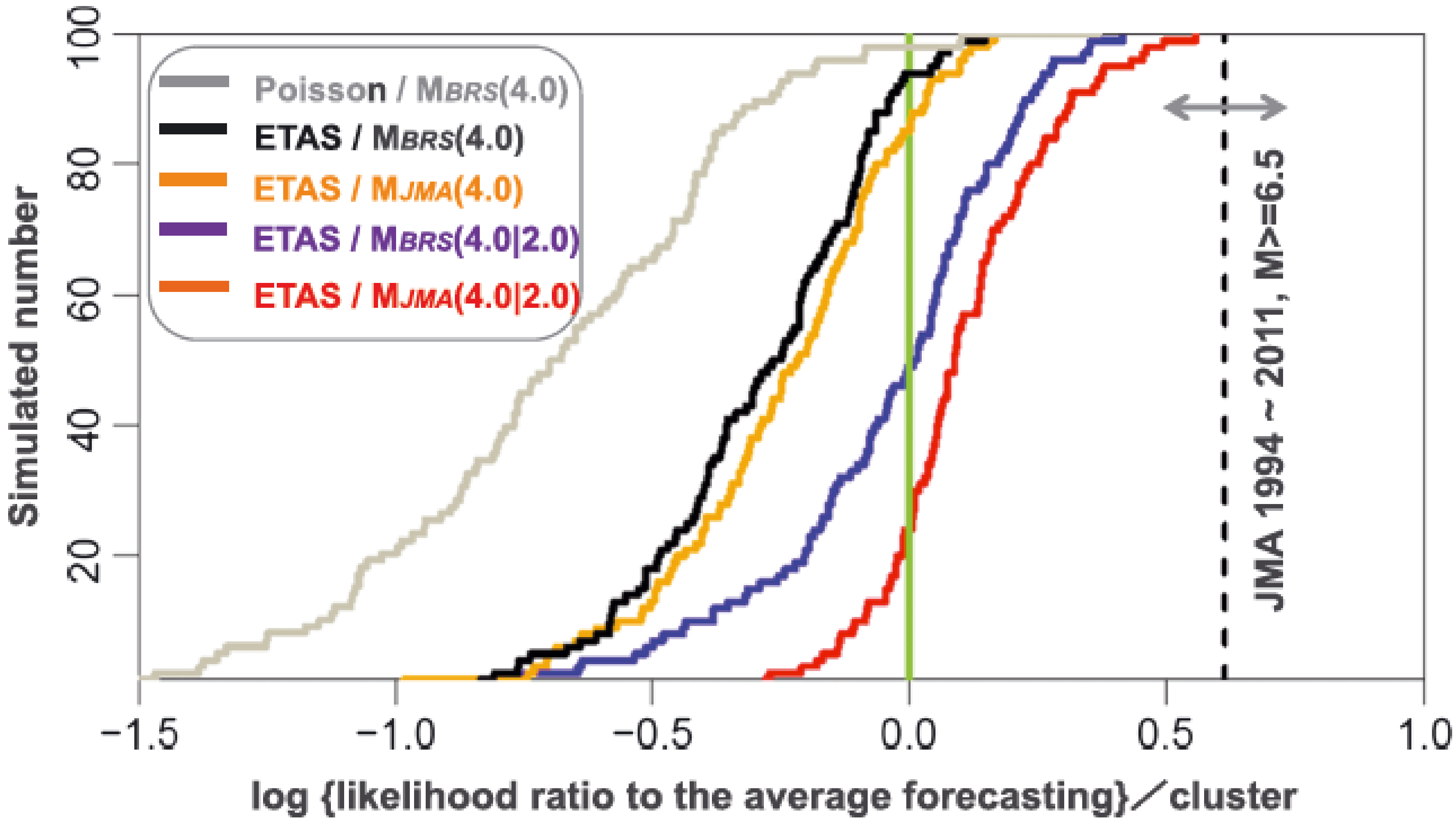
mean score per cluster for $(1/N)\ln(L_1/L_0) = \sum_{c=1}^{N} Q_c/N$ is 0.0362 ± 0.00826.

Foreshock forecast evaluations in case where multiple earthquakes observed in a cluster



$$\frac{\log\{likelihood\ ratio\}}{number\ \ of\ \ clusters} = \frac{1}{\#c}\ln\left\{\frac{L_1}{L_0}\right\} = \frac{1}{\#c}\sum_c\left\{\eta_c \ln\frac{p_c^{\eta_c}}{\bar{p}} + (1-\eta_c)\ln\frac{1-p_c^{\eta_c}}{1-\bar{p}}\right\}$$

mean score per cluster for $(1/N)\ln(L_1/L_0) = \sum_{c=1}^{N} Q_c/N$ is 0.0362 ± 0.00826.



$$\frac{\log\{likelihood\ ratio\}}{number\ \ of\ clusters} = \frac{1}{\#c}\ln\left\{\frac{L_1}{L_0}\right\} = \frac{1}{\#c}\sum_c\left\{\eta_c \ln\frac{p_c^{\eta_c}}{\overline{p}} + (1-\eta_c)\ln\frac{1-p_c^{\eta_c}}{1-\overline{p}}\right\}$$

JMA; Main shock M ≧ 6.5　　22.74／37 = 0.61

Summary on stacking characteristics

1. Stacked foreshocks in the synthesized catalogs have weaker concentration than those in real earthquake catalog such as the JMA catalog.

2. Such quantitative features depend on the mainshock size, and have different characteristics between the ETAS catalogs and the JMA catalog.

3. Some differences of foreshock features are partly due to the generating scheme of magnitude sequence.

Summary on foreshock predictors

1. The foreshock predictor show similar trend in probability gain to the average foreshock probability over all catalogs including Poisson catalog.

2. The **log-likelihood-ratio scores per cluster** of the predictor vary in different ranges depending on the generation scheme of magnitudes and ETAS, and the score in JMA catalog are larger than the synthetic catalogs.