

Critical zone of the branching crack model for earthquakes: inherent randomness, earthquake predictability, and precursor modelling

Jiancang Zhuan¹, Mitsuhiro Matsu'ura¹, & Peng Han²

¹*Institute of Statistical Mathematics, Research Organization of Information and Systems, Tokyo, Japan*

²*Department of earth and space sciences, Southern University of Science and Technology, Shenzhen, China*

The branching crack model was developed by Vere-Jones and Kagan in 1970s and 1980s. With some natural assumptions, its simulation results explain the Gutenberg-Richter magnitude-frequency relationship and the Omori-Utsu formula. By assuming that each generation is a time step and that the seismic moment released at each time step is the number of cracks, Zhuang et al. (2016) emulated the source-time function in by using this model. Such source-time functions are quite similar to source-time functions after smoothing, exhibiting single or multiple peaks and no fixed shapes. If the branching process does not stop at a certain time step, any number of cracks or peaks are possible to be produced in its continuation. Such inherent randomness explains why the earthquake magnitude cannot be determined unless the recorded waveforms contain information of the entire rupture process and why earthquakes cannot be predicted in a deterministic way or with almost-sure certainty. By introducing the critical zone and other related concepts, this model can be connected to the asperity, the barrier and the nucleation models through a parameter -- criticality. Particularly, the critical zone determines the potential maximum magnitude of the future earthquakes and the source of anomalies. From the viewpoint that the key-point for earthquake forecasting is to determine whether an area is in a critical state and how large the critical zone is, we discuss what anomalies are meaningful as candidates of earthquake precursors. Finally, we outline modelling strategies for earthquake precursors when probability gains are low due to the inherent randomness of the earthquake source process. We conclude:

(a) The biggest difference of the real world from the critical branching crack process is that, in the real world, only the critical zone with finite volume can be in a state of critical but not the whole crust. The size of the critical zone gives the possible maximum magnitude of future earthquakes.

(b) The critical zone can be detected by its stress field and other phenomena. Among many proposed earthquake precursors, we believe that acceleration of micro-seismicity, the b -value, the LURR, the criticality parameter in the ETAS model, are good indicators of the current critical level. Other observations, such as GPS displacements, gravity field changes, and electromagnetic field changes are shown useful, even though the probability gains are lower than our anticipated.

(c) Due to the inherent randomness of the cracking process, these precursory indicators have an upper limit of probability in forecasting, which may be a bit far from satisfactory. High performance forecasts can be and possibly can only be made based on multidisciplinary precursors.

(d) As the clustering effect is the biggest predictable component in seismicity, the modelling of the explanatory effect of the anomalies to earthquakes should be constructed from the ETAS model, as done in Han et al (2016) 's ETAS model with external excitations.

In summary, we are still optimistic to earthquake forecast. The main task is to develop monitoring technologies that can help us to detect effective precursory anomalies and to determine the size of the critical zone and the critical status of the area of interests. Moreover, developments of statistical inference and modelling methods for multidisciplinary precursors are also indispensable.

References

- Chen, S., C. Jiang, and J. Zhuang (2016), Statistical evaluation of efficiency and possibility of earthquake predictions with gravity field variation and its analytic signal in western China, *Pure and Applied Geophysics*, 173 (1), 305--319, doi:10.1007/s00024-015-1114-x .
- Geller, R. G., D. D. Jackson, Y. Y. Kagan, and F. Mulargia (1997) Earthquakes cannot be predicted. *Science*, 275, 1616--1617.
- Han, P., K. Hattori, M. Hirokawa, J. Zhuang, C. H. Chen, F. Febriani, H. Yamaguchi, C. Yoshino, J. Liu, and S. Yoshida (2014) Statistical analysis of ULF seismomagnetic phenomena at Kakioka, Japan, during 2001—2010. *Journal of Geophysical Research: Space Physics*, 119 (6), 4998--5011, doi:10.1002/2014JA019789.
- Han, P., J. Zhuang, K. Hattori, and Y. Ogata (2016) An interdisciplinary approach for earthquake modelling and forecasting. AGU Fall Meeting Abstracts, S23E-03.
- Han, P., K. Hattori, J. Zhuang, C.-H. Chen, J.-Y. Liu, and S. Yoshida (2017) Evaluation of ULF seismo-magnetic phenomena in Kakioka, Japan by using Molchan's error diagram, *Geophysical Journal International*, 208 (1), 482--490, doi:10.1093/gji/ggw404 .
- Jordan, T., Y.-T. Chen, P. Gasparini, R. Madariaga, I. Main, W. Marzocchi, G. Papadopoulos, G. Sobolev, K. Yamaoka, and J. Zschau (2011) Operational earthquake forecasting. state of knowledge and guidelines for utilization. *Annals of Geophysics*, 54 (4), doi:10.4401/ag-5350 .
- Kagan, Y. Y. (1982). Stochastic model of earthquake fault geometry, *Geophys. J. Roy. Astr. Soc.*, 71, 659--691.
- Kagan, Y. Y. (2010). Earthquake size distribution: Power-law with exponent $\beta \equiv 1/2$. *Tectonophysics*, 490 (1-2), 103 – 114, doi:10.1016/j.tecto.2010.04.034 .
- Nanjo, K. Z., N. Hirata, K. Obara, and K. Kasahara (2012), Decade-scale decrease in b value prior to the M9-class 2011 Tohoku and 2004 Sumatra quakes. *Geophysical Research Letters*, 39 (20), doi:10.1029/2012GL052997.
- Ogata, Y. (1988) Statistical models for earthquake occurrences and residual analysis for point processes. *Journal of the American Statistical Association*, 83 (401), 9--27, doi:10.1080/01621459.1988.10478560 .

- Ogata, Y., and J. Zhuang (2006) Space-time ETAS models and an improved extension. *Tectonophysics*, 413 (1-2), 13--23.
- Rydelek, P., and S. Horiuchi (2006) Earth science: Is earthquake rupture deterministic?. *Nature*, 442, E5--E6, doi:10.1038/nature04963 .
- Vere-Jones, D. (1976) A branching model for crack propagation. *Pure and Applied Geophysics*, 114 (4), 711--725, doi:10.1007/BF00875663 .
- Vere-Jones, D. (1977) Statistical theories of crack propagation. *Journal of the International Association for Mathematical Geology*, 9(5), 455--481, doi:10.1007/BF02100959 .
- Wang, T., J. Zhuang, T. Kato, and M. Bebbington (2013) Assessing the potential improvement in short-term earthquake forecasts from incorporation of GPS data. *Geophysical Research Letters*, 40(11), 2631--2635, doi:10.1002/grl.50554 .
- Yin, X., X. Chen, Z. Song, and C. Yin (1995) A new approach to earthquake prediction-the load/unload response ratio (LURR) theory. *Pure and Applied Geophysics*, 145(3-4), 701--715.
- Zhuang, J. (2011) Next-day earthquake forecasts by using the ETAS model. *Earth, Planet, and Space*, 63, 207--216, doi:10.5047/eps.2010.12.010 .
- Zhuang, J., Y. Ogata, and D. Vere-Jones (2002) Stochastic declustering of space-time earthquake occurrences. *Journal of the American Statistical Association*, 97 (3), 369--380.
- Zhuang, J., D. Vere-Jones, H. Guan, Y. Ogata, and L. Ma (2005) Preliminary analysis of observations on the ultra-low frequency electric field in a region around Beijing. *Pure and Applied Geophysics*, 162, 1367--1396, doi:10.1007/s00024-004-2674-3.
- Zhuang, J., Y. Ogata, D. Vere-Jones, L. Ma, and H. Guan (2013) Statistical modeling of earthquake occurrences based on external geophysical observations: with an illustrative application to the ultra-low frequency ground electric signals observed in the Beijing region, in *Imaging, Modeling and Assimilation in Seismology*, Vol. II, edited by Y. Li, pp. 351--377, De Gruyter, Germany.
- Zhuang, J., D. Wang, and M. Matsu'ura (2016) Features of earthquake source process simulated by vere- jones' branching crack model. *Bulletin of the Seismological Society of America*, 106 (4), 1832, doi:10.1785/0120150337 .