

## **A fractal study of self-potential time series.**

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### **Abstract**

In the current literature on seismo electromagnetic has been reported many earthquakes which present electromagnetic anomalies as probable precursors of their occurrences. Although this methodology remains yet order discussion, it is considered interesting the study of many particular cases. In this work we report a fractal analysis of electroseismic signals recorded in the Acapulco station during 1993. In October 24, 1993 occurred and earthquake (EQ) with M 6.5 an epicenter (16.54 N, 98.98 W), 100Km a way the mentioned station. Here we calculate the so called Higuchi's fractal Dimension and the Hurst exponent to signals during one month before the EQ. We discuss the dynamical meaning of this analysis and its possible relation with the mentioned EQ.

### **1. INTRODUCTION**

In many seismically actives zone around the world there exist research programs for the study of precursory phenomena of seisms (Lomnitz, 1990; Rikitake, 1976; Hayakawa, 1999). One of the techniques used in the search of phenomena precursors of seisms since more twenty seven years ago consists in monitoring the so-called electric self-potential field. This field is studied through measurements of the ground electrical potential (the self-potential) by means of shallow pairs of unpolarized electrodes buried in the ground generating a voltage time series  $\Delta V = \Delta V(t)$ . several authors have proposed a correlation between patterns of self-potential variations and the mechanism of preparation of earthquakes (Varotsos and Alexoupoulos., 1948a; 1948b).

For some years, we have taken registers of the fluctuations of electric self-potential of the ground in several sites of Mexico (Yépez et al., 1995; 1999), these registers were taken by means of electric self-potential stations

Some stations are located along the coast of Guerrero state, near the Middle American Trench, which is the border between the Cocos and the American tectonic plates. In this work, we study the electric self-potential time series arising from station of Acapulco (16.85 N, 99.9 W) linked to the Middle American trench. In this typical station, thousands of data are taken each two (or four) seconds during periods in the scale of months and years.

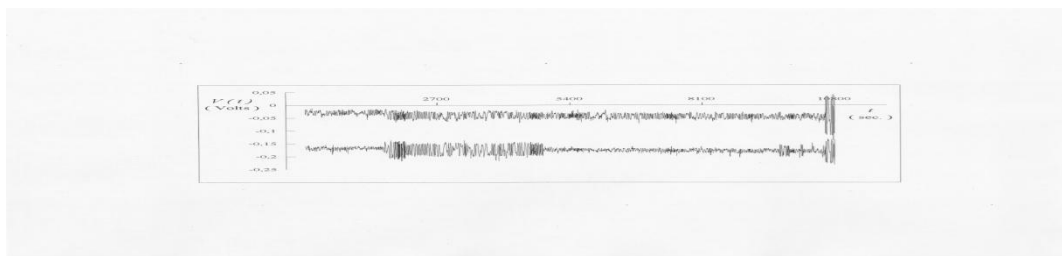


Fig. 1. Typical segments of self-potential time series collected at Acapulco station. The superior series is the N-S one and the down series correspond to the E-W line. Here two 5hr series are showed, with a sampling period of  $\tau$  seconds. The self-potential time series corresponding to a 2 hr-file.

## 2. Scaling Law and Fractal Dimension

These time series have been analyzed by means of several methods as the spectral ones (Yépez et al., 1995; 1999). In this paper, we report a study of our electric self-potential time series by means of the fractal dimension  $D$  by using the Higuchi's algorithm (Higuchi., 1988).

In our study of electric self-potential time series, we use the Higuchi's algorithm for calculating their fractal dimension  $D$ . Usually, the exponent  $\beta$  is considered to be the index for representing the irregularity of a time series, although the fractal dimension  $D$  can be also used as index of irregularity. In fact, in many cases the usage of  $D$  is more appropriate than the spectral exponent  $\beta$  for determining irregularity indices (Cervantes et al., 1999). The fractal technique developed by Higuchi (1988) gives stable indices even for a small number of data. Higuchi (1988; 1990) considers a finite set of time series observations taken at a regular interval:  $\Delta V(1), \Delta V(2), \Delta V(3), \dots, \Delta V(n)$ . From the given time series, he first constructs a new time series,  $\Delta V_k^m$ , defined as follows.

$\Delta V_k^m: \Delta V(m), \Delta V(m+k), \Delta V(m+2k), \dots, \Delta V(m + [\frac{n-k}{k}], k)$ , with  $m = 1, 2, \dots, k$ , and where  $[ ]$  denotes the Gauss notation, and  $m, k$  are integers that indicate the initial time and the time interval respectively. For a time interval equal to  $k$ , one gets  $k$  sets of new time series. Higuchi defines (1988) the length of the curve associated to each time series  $\Delta V_k^m$  as follows:

$$L_{m(k)} = \frac{\sum_{i=1}^{\lfloor \frac{N-m}{k} \rfloor} [\Delta V(m+ik) - \Delta V(m+(i-1)k)] \left( \frac{N-1}{\lfloor \frac{N-m}{k} \rfloor k} \right)}{k} \quad (1)$$

where the term  $\frac{(N-1)}{\lfloor \frac{N-m}{k} \rfloor k}$  represents a normalization factor, then the length of the curve for the time interval  $k$  is taken as the average value  $\langle L(k) \rangle$  over  $k$  sets of  $L_m(k)$ . If the average value obeys the following scaling law:

$$\langle L(k) \rangle \propto k^{-D}, \quad (2)$$

then the curve is fractal with dimension  $D$  (Higuchi, 1988). This algorithm can be applied even over time series that are not stationary and this fact represents an advantage over the spectral techniques (Cervantes et al., 1999). Higuchi (1990) shows that if  $1 \leq \beta \leq 3$  then  $D = \frac{5-\beta}{2}$  (equation (2)) is held, he also shows that the following limits are held,

$$\text{if } \beta \rightarrow 0 \text{ then } D \rightarrow 2, \quad (3)$$

which corresponds to uncorrelated white noise, while the second limit is: if  $\beta \rightarrow 3$  then  $D \rightarrow 1$ .

Since the values of the fractal dimension  $D$  for our time series are in the interval [1,2], the equations (1) and (2) can be used for their analysis. In figures 1a and 1b we show typical  $\Delta V(t)$  electric self-potential time series for N-S and E-W electrode pairs respectively. For the calculation of the fractal dimension  $D$ , by means of the Higuchi algorithm we divide the  $\Delta V(t)$  time series data in 6 hrs-files (10800 V-points). To each file, we associate a fractal dimension  $D$  taken from (2), calculating the slope of the double log plot of  $\langle L(k) \rangle$  against  $k$ . Applying the Higuchi's algorithm to data of 1993. It is convenient to remark that in the studied series the calculation of the fractal dimension  $D$  is statistically appropriate; that is, the correlation coefficient  $R^2$  is near to one, and the associate errors of the fractal dimension  $D$  are worthless. That is, the obtained graphs of  $\log \langle L(k) \rangle$  vs.  $\log k$  for our electric self-potential time series are always statistically straight lines.

The Hurst exponent:

The description developed by Harold Hurst himself is as follows:

$$E[R(n)/S(n)] = C n^H, \quad (4)$$

The left hand side is also known as the expected value of the rescaled range (Hurst, ).  $R(n)$  is defined on a time  $\Delta V_i$ ,  $i=1, 2, \dots, n$  as follows

$$R(n) = \text{Max}(\Delta V_i) - \text{Min}(\Delta V_i) \quad (5)$$

$S(n)$  is the standard deviation and "C" an arbitrary constant.

### 3. Some Results for Self-Potential Time Series.

In this work, for our electric self-potential time series, we have calculated the following invariant

quantities: Fractal Dimension  $D$  (Higuchi's Algorithm) and the Hurst exponent from the rescaled range.  
 For the Acapulco station we study the behavior of the Fractal Dimension from 1993; we study the temporal evolution of the fractal Dimension  $D$  (figure 2).

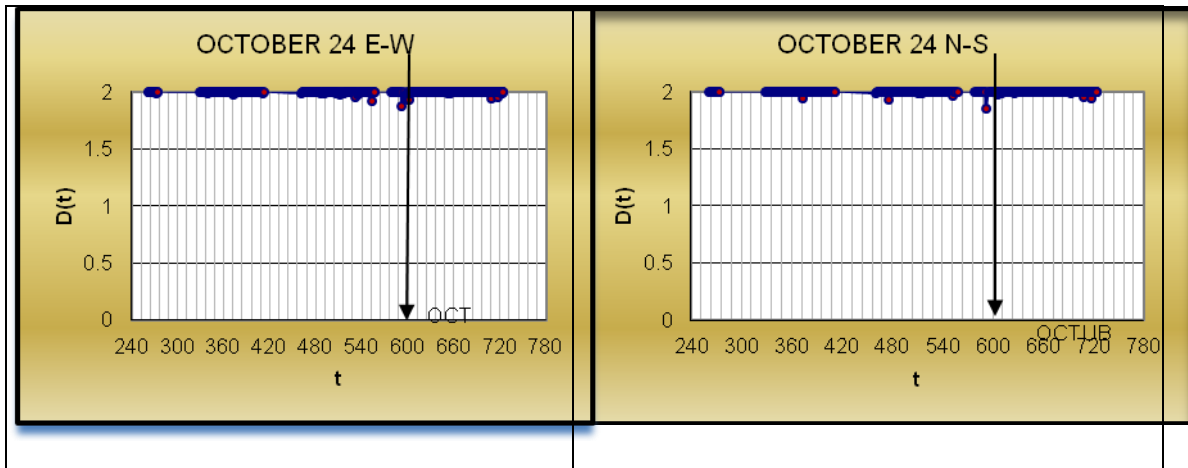
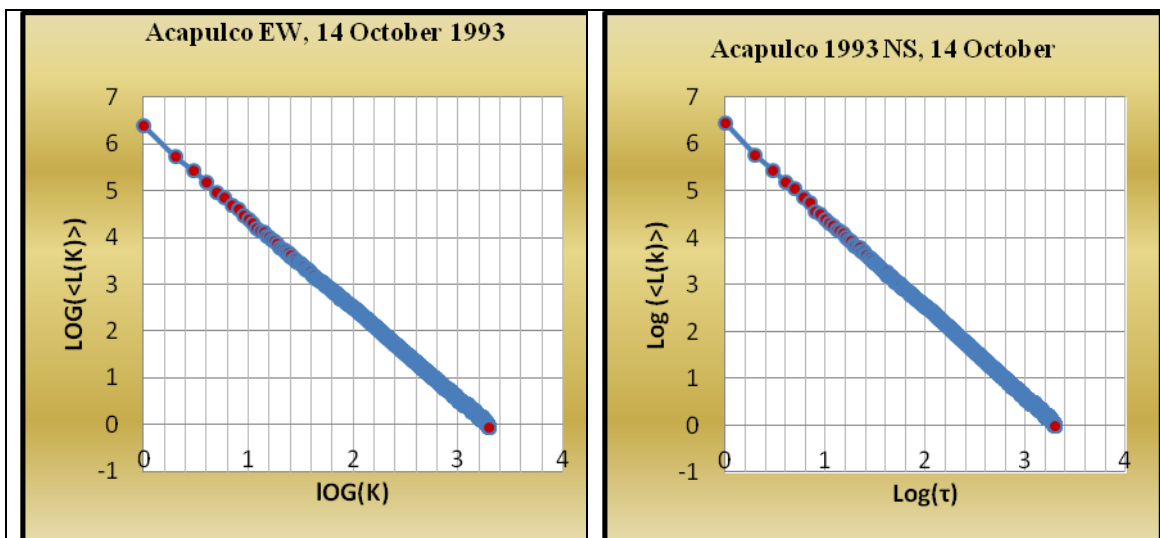
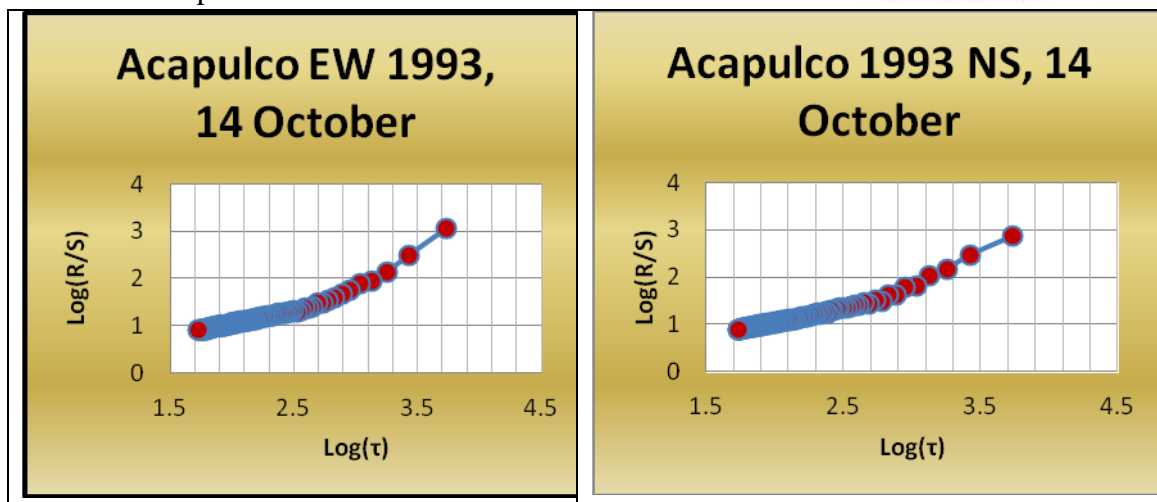


Fig. 2 Time evolution of fractal dimension  $D$ , 1993.

The October 24, 1993 earthquake of  $M_w=6.6$  occurred. In October 14, 1993, an anomalous behavior of the fractal dimension  $D$ , and of the Hurst exponent





#### 4. Concluding remarks

By means of electrotelluric time series taken from the Acapulco station, near of Middle American trench, which is a very seismically active zone, we study the dynamical behavior of the fractal dimension. We have to find that the critical behavior of the fractal dimension ( $D \leq 1.8$ ) to be able is correlated with the occurrence the seisms with magnitude greater or equal than  $M_w 5.8$ .

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